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Why is the Universe Described in Mathematics?

This essay attempts to address the questions, "Why is the Universe described in mathematics?" and "Is mathematics a human creation, specific to our Universe, or equally valid for all Universes?"

To answer this question, we first need to define mathematics. Modern algebraic mathematics can be thought of as having two components: a set of axioms, and all statements which logically follow from these axioms. To make progress in mathematics, one gives a proof which combines the axioms together with logical arguments to arrive at a conclusion. There have been difficulties in trying to formalize exactly what is meant by this – ideally, a perfect formal theory of mathematics would allow all mathematicians and computers to check any purported proof and agree on whether the proof is correct or not. It is straightforward to express all of the axioms, conclusions, and steps in a proof in the most formal of languages. However, the difficulty in formalization lies in the deduction process which is used to check the correctness of proofs.

If mathematics is to be truly formal and on "firm ground," rather than being intuitive, then it must be possible to express the deduction process itself as a purely mechanical or computational process. This mechanical process is typically also described in the language of mathematics, so a great deal of confusion immediately ensues on what is meta-mathematics (used to describe the "proof checker" that mathematicians and computers alike use to check proofs) and what is mathematics (the actual assumptions that one is using as "input" to the "proof checker"). As soon as there is a system of "meta-mathematics" which allows logical deduction *ex nihilo* separate from the system of "mathematics" which allows logical deduction from only its own axioms, it would seem that the quest to formalize mathematics has wholly failed, as mathematics can only be expressed and analyzed in its own language, so we have tautology, a recursive problem. In practice, mathematicians skirted around this problem by assuming that "meta-mathematics" or logical inference "just works." Typically, either the logical faculties of the mathematician reading the paper, or the first- or second-order logics are used as the "meta-mathematics;" however, the underlying idea is that any mathematician simply must accept as an article of faith that logical deduction "works." With the logical inference process fixed, mathematicians are then allowed to change the axioms in everyday mathematics arbitrarily, and see what results follow. This approach has been used largely as the foundation of modern math – for example, in 1910, Bertrand Russell and Alfred Whitehead published the influential book *Principia Mathematica* which on page 379 shows that one plus one equals two[1]. This may seem silly, yet the strength of such formalization is that it unifies mathematicians. If a group of people can agree on the same mathematical axioms, then these people will most certainly use the same rules for logical inference, and so notwithstanding human error, then this group of people will always agree on the validity or invalidity of a given mathematical proof. For a modern and poignant example of this, one can consult the Metamath website[2], which uses the Zermelo-Fraenkel axioms with the axiom of choice (ZFC), ZFC with added Hilbert space axioms, and quantum logic axioms to develop 6,600 proofs for much of modern mathematics. It should be noted that additional axioms must be incorporated in the process of making such proofs, such as the axioms of real and complex numbers.

Remarkably, a 275 line proof checker for the Metamath proof language was independently written by computer programmer Raph Levein[2].

At this point, one can ask a deeper question, "If physics requires mathematics, and mathematics requires logical deduction, is logical deduction itself valid, or still merely a human creation?" Strictly speaking it is impossible to defend logical deduction with any argument, because logic must be used in the argument of defense. However, it seems quite reasonable to expect that in any conceivable Universe, the same rules of logical deduction would hold. Physics may behave in seemingly illogical ways in other Universes, yet deduction itself seems to be such a fundamental thing that it is above philosophy, even, as it is used in the reasoning of philosophy.

If mathematics is merely a way of stating arbitrary assumptions and writing proofs from these assumptions, then one has to wonder why mathematics is suitable for expressing the way the Universe works, i.e. physics. That is, is mathematics a human creation, and subject to human whims and "axiom fads?" Or is there something intrinsic about mathematics so that only "one" mathematical axiom set can be naturally developed by humans, and this "one" mathematics is also suitable for describing the Universe? To address this question we can look at the abstractions we have developed: calculus, complex numbers, probability, differential geometry, topology, and so forth. Many of these abstractions have the quality that they are somehow the most "natural," and the most "simple" way that an idea can be expressed and analyzed. For example, calculus is the study of areas and slopes; complex numbers are the answer to the question of whether polynomials over the reals can be factored, and what would be "needed" to factor these polynomials. Differential geometry is the study of surfaces which can be differentiably bent into planes, and planes can be seen as a Platonic ideal. It hardly seems that humans have developed mathematics randomly, accepting whichever axioms are popular in the day. Rather, it seems that mathematics has developed very carefully from the study of the simplest things first.

Yet why is mathematics the language of physics? It is undeniable that our current physical models are not correct, so perhaps mathematics is not the language of physics. The physics community today is stumped because no one can figure out how to correctly model quantum gravity. Physicists are searching for the theory of quantum gravity, which many presume will be a "theory of everything." This may be overly optimistic. It is possible that the process of replacing old theories with new theories may never end – each time we account for some new experimental evidence there will be an additional problem in the physical theories, so that the "correct" physics will never be found. In the language of mathematics, the set of axioms necessary for physics may have too large a cardinality for humans to discover, even in principle.

However, the relatively simple laws of Newtonian mechanics, general relativity, and quantum mechanics are remarkably accurate for describing phenomena in their appropriate regimes. And these simple laws coincide amazingly well with the sort of "natural" development of mathematics – the progression in mathematics which I mentioned earlier where the simplest analyzable ideas are investigated first. With a little imagination, one can imagine that the laws of the Universe are a sequence of stepping stones, which have been set up beforehand so that the natural progress of mathematics leads up the steps.

I would argue that the stepping stones of understanding physics largely have to do with understanding geometry. The laws of classical physics and general relativity are clearly geometrical. Symmetry is used extensively in particle physics. Baryons are arranged in multiplets according to quark number. As noted by Barrow in the essay "Why is the Universe Mathematical?," conservation laws for elementary particles are described by group theory[3]. While I do not mean geometry in the simple Euclidean sense, it seems silly to think that our mathematics symbols are fundamentally what the Universe "is," so geometry here is an attempt to generalize the abstract ideas of warped spaces, finite spaces, symmetric spaces, and so forth. From this perspective, there is nothing "real" at all about the mathematics we use; the representation of physics by the geometries we have so far discovered is purely an abstraction, and the actuality is that the world keeps its own geometry, and we are looking at its pre-existing geometry from one particular vantage – that of naivety – and describing it in one particular language – the language of modern algebra, of forces, path integrals, fields, and space-time curvature. This perspective doesn't seem to work quite as well for quantum mechanics, as quantum mechanics is based on the spatial distributions of wavefunctions, axioms of probability, complex numbers, and differential equations. That is, in quantum mechanics, at first glance, it seems easier to just think of the math, and there doesn't seem to be any underlying geometry or beauty or symmetry "beneath" the math. Indeed, one peculiarity of quantum mechanics which confounds physicists is that one often uses symbols without any knowledge of the real meaning of the symbols. However, one can still argue that quantum mechanics is based on spatial geometry, because wavefunctions are a distribution throughout space, and although complex numbers are not intuitive, they are a vector space over the reals and a field, and operations such as multiplication have geometrical interpretations as rotation and scaling.

Thus my argument is that the Universe probably has a complex geometry, coupled with some axiomatic laws needed to deduce the rest of physics. The Universe's geometry is analogous to that of a snowflake: it has a number of principle components which are simpler. The simplest approximations are 3 space dimensions (classical mechanics), followed by 4 curved spacetime dimensions (general relativity), 4 flat spacetime dimensions (quantum field theory with special relativity), 11 spacetime dimensions (superstring theory), followed by a finite or infinite sequence which eventually terminates with the true geometry of the Universe. The human mathematical notation merely attempts to capture the aspects of this geometry which we can observe, and we may never be able to capture the subtleties of the Universe's true geometry if it has an infinite number of dimensions or extremely "small" or topologically bizarre dimensions. However, this perspective helps clarify that humans are not inventing anything by creating the language of mathematics or developing concepts such as the field or the path integral; instead we are trying to correctly describe the pre-existing geometries of space and matter.

Yet one wonders why naive mathematical exploration yields so readily the nearly universal laws of physics. For example, the Universe could have non-integer dimension, or have a large spacetime dimension such as 10^{500}. I don't know how to answer this question; if one knew the final theory of everything, then the progression from Newton's laws to quantum mechanics and onward might be more clear, but today it seems that the Universe is simply benevolent – it wants to be discovered.

Perhaps more importantly, not all Universes can even be described with mathematics. There are two archetypal counterexamples: a Universe with randomness, and a Universe which not self-consistent. In the former case, the unpredictable Universe cannot be modeled with mathematics (except via probability, which really measures a lack of knowledge and an inability to predict the future). By "self-consistent," I mean that if two events happen in different places in the Universe, then they will unfold by the same underlying physical laws. In a Universe which is not self-consistent, it could be very difficult to do experimental science, as the outcomes of experiments would change based on one's position, the time of day, and so forth. If a Universe follows a global set of laws but is not self-consistent, then in theory humans could develop mathematics to describe the global laws, yet in practice this might be impossible due to the difficulty in obtaining meaningful results from experiments. Despite the "spooky action-at-a-distance" character of quantum mechanics, at human length scales our Universe is self-consistent: if I drop a ball at any position on the Earth, it will fall to the ground. The ball will not follow confusing paths due to some global laws which humans do not know, and the ball will not follow a random path. Even quantum mechanics is self-consistent, because the same laws of quantum mechanics are used "locally" at every position in the Universe.

In conclusion, I would argue then our Universe has an underlying geometric character which can be successively approximated by geometries in three, four, or eleven dimensions. I would also argue that our Universe is self-consistent and on the macroscopic level non-random. These characteristics allow us to develop mathematical theories which describe the Universe. Unsurprisingly, the human development of mathematics started from the simplest and most naive concepts of the natural numbers, the integers, the reals, and progressed to more complex ideas such as calculus, the complex numbers, geometry, group theory, and so forth. Each of these ideas is analyzable in the form of equations; thus mathematics can be said to have a natural progression, and is not merely an arbitrary cloud of axioms. Yet mysteriously, this natural progression in mathematics had applicability in physics. This lends credence to the idea that there is great similarity between the Universe and axiomatic logic, and that the Universe's laws are "close" to the simplest laws which will be found by any sentient being investigating axiomatic logic. Thus if we encounter aliens, I could expect them to have developed the laws of physics using different mathematical notation, yet with the same order of discovery of the same mathematical ideas. One can imagine alternative Universes which have random behavior or are not self-consistent. It would be difficult to use mathematics to describe such Universes; however, by doing simple experiments such as dropping similar objects repeatedly, we observe that our Universe in the classical regime is both predictable and self-consistent. This is interesting, because it indicates that even the ancient Greeks could have predicted that the large-scale Universe for the most part follows mathematical laws, though the exact description of the laws would not have been apparent without further experiments. By doing experiments, we can find that the subatomic world is unpredictable and self-consistent. These characteristics allow us to use mathematics to describe the Universe.

Bibliography

[1]. Russell, Bertrand and Whitehead, Alfred. *Principia Mathematica, Vol* 1. 1910. Cambridge University Press. Available: http://www.hti.umich.edu/cgi/t/text/text-idx?c=umhistmath;idno=AAT3201.0001.001

[2]. Megill, Norman. *Metamath Home Page*. 1996-2006. Available: http://us.metamath.org/

[3]. Barrow, John. *Between Inner Space and Outer Space*. 1999. Oxford University Press, USA.