

Cosmology HW#3 Problem #1

a) A Geodesic would be the path followed by a test particle under the influence of gravity only. The rocket follows a geodesic after the rocket engines have turned off. The clock on earth is subject to the reaction force of the earth. The earth's clock counted the smaller amount of time.

b) As discussed in class, the appropriate time integral is

$$\tau_{AB} = \int_A^B \left[1 - \frac{1}{c^2} (v^2/2 - \phi) \right] dt$$

Here are some simplifying assumptions:

1. The acceleration phase required an negligible amount of time.
2. The maximum height of the rocket $\ll R$, the radius of the earth.

Newton says: $r = R + v_0 t - \frac{1}{2} g t^2 =$ distance from center of earth
 $v = v_0 - g t$

$$\phi(r) = - \frac{MG}{r} = \frac{-MG}{R + v_0 t - \frac{1}{2} g t^2}$$

$$= - \frac{MG}{R(1 + \delta(r))} \approx -Rg(1 - \delta(r))$$

where $\delta(r) = \frac{v_0 t - g t^2 / 2}{R}$

$$\tau_{AB} = \int_0^T \left[1 - \frac{1}{c^2} \left(\frac{1}{2}(v-gt)^2 + Rg(1-gt) \right) \right] dt$$

$$= T - \frac{TgR}{c^2} - \frac{T}{6c^2} (2g^2T^2 - 6v_0gT + 3v_0^2)$$

The first two terms would also affect the earth-bound clock, so

$$\Delta \tau = -\frac{T}{6c^2} (2g^2T^2 - 6v_0gT + 3v_0^2)$$

where $T = \sqrt{2v_0/g}$

2. we regard a as a fixed scale factor

In Euclidean 3-space $ds^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$

So here $dx^2 + dy^2 + dz^2 = a^2 [dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)]$

$$\text{also } w^2 = a^2 - r^2 a^2 \quad w = a \sqrt{1 - r^2}$$

$$dw = -\frac{a r dr}{\sqrt{1 - r^2}} \quad dw^2 = \frac{a^2 r^2 dr^2}{1 - r^2}$$

Then

$$\begin{aligned} ds^2 &= dw^2 + dx^2 + dy^2 + dz^2 \\ &= a^2 \left[\frac{dr^2}{1 - r^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right] \end{aligned}$$

which is the R-W metric in cylindrical coord.

