Cosmology HW # 3 Problem # 1

a) a Geodesic would be the path follow by a test particle under the influence of gravity only. The rocket follows a geodesic after the rocket engines have turned off. The clock on earth is subject to the reaction force of the earth. The earth's clock counted the smaller amount of time.

b) As discussed in class, the appropriate time integral is

$$\tau_{AB} = \int_A^B \left[1 - \frac{1}{c^2}\left(v^2/_2 - \Phi\right)\right] dt$$

Here are some simplifying assumptions:

1. The acceleration phase required an negligible amount of time
2. The maximum height of the rocket << R, the radius of the earth.

Newton says: $r = R + v_0 t - \frac{1}{2} g t^2$ = distance from center of earth

$v = v_0 - g t$

$$\Phi(r) = -\frac{MG}{r} = \frac{-MG}{R + v_0 t - \frac{1}{2} g t^2}$$

$$= -\frac{MG}{R(1+\delta(r))} \simeq -Rg\left(1 - \delta(r)\right)$$

where $\delta(r) = \dfrac{v_0 t - g t^2/2}{R}$

$$\tau_{AB} = \int_0^T \left[1 - \frac{1}{c^2}\left(\frac{1}{2}(v - gt)^2 + Rg\,(1 - \delta(t)) \right) \right] dt$$

$$= T - \frac{TgR}{c^2} - \frac{T}{6c^2}\left(2g^2T^2 - 6v_0 gT + 3v_0^2 \right)$$

The first two terms would also affect the earth-bound clock, so

$$\Delta\tau = -\frac{T}{6c^2}\left(2g^2T^2 - 6v_0 gT + 3v_0^2 \right)$$

where $T = \sqrt{2v_0/g}$

2. we regard a as a fixed scale factor

In Euclidean 3-space $ds^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$

So here $dx^2 + dy^2 + dz^2 = a^2\left[dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$

also $w^2 = a^2 - r^2 a^2$ $\qquad w = a\sqrt{1-r^2}$

$$dw = -\frac{a r\, dr}{\sqrt{1-r^2}} \qquad dw^2 = \frac{a^2 r^2\, dr^2}{1-r^2}$$

Then

$$ds^2 = dw^2 + dx^2 + dy^2 + dz^2$$

$$= a^2\left[\frac{dr^2}{1-r^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

which is the R-W metric in cylindrical coor.

3. If $\chi = \pi$ the R-W metric is

$$ds^2 = -dt^2 + a^2 d\chi^2$$

photons travel along the path $ds^2 = 0$ $dt^2 = a^2 d\chi^2$
all paths are geodesics, regardless of θ & ϕ.