Cosmology Problem Set #5

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The following exercises are due Monday, March 12.

1. In class I explained how to measure the mass in galaxies and galaxy clusters using the virial theorem. The basic formula is

$$M = \frac{\langle v^2 \rangle r_h}{\alpha G},$$

where r_h is the half-mass radius and α is a parameter determined by data fitting. For our purposes $\alpha \approx 0.4$.

The Draco galaxy is a dwarf galaxy within the Local Group. Its luminosity is $L = 1.8 \times 10^5\ L_\odot$ and half its total luminosity is contained within a sphere of radius $r_h = 120$ pc. The red giant stars in the Draco galaxy are bright enough to have their line -of-sight velocities measured. The measured velocity is 31.5 km s^{-1}. What is the mass of the Draco Galaxy? What is the mass-to-light ratio? Given the fact that typical stars have a mass-to-light ratio of $4M_\odot/L_\odot$, what fraction of the galaxy's mass is dark matter?

2. One of the more recent speculations in cosmology is that the universe may contain a quantum field, called "quintessence," which has a positive energy density and a negative value of the equation-of-state parameter w. Assume, for the purposes of this problem, that the universe is spatially flat, and contains nothing but matter ($w = 0$) and quintessence with $w = -1/2$. The current density parameter of matter is $\Omega_{m,0} \leq 1$, and the current density parameter of quintessence is $\Omega_{Q,0} = 1 - \Omega_{m,0}$. At what scale factor a_{mQ} will the energy density of quintessence and matter be equal? Solve the Friedman equation to find $a(t)$ of the universe. What is $a(t)$ in the limit $a \ll a_{mQ}$? What is $a(t)$ in the limit $a \gg a_{mQ}$? What is the current age of the universe, expressed in terms of H_0 and $\Omega_{m,0}$?