Cosmology Problem Set #4

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The following exercises are due Monday, February 26.

1. It's possible to have a completely empty, curved universe. Solve the Friedman equation and find $a(t)$ as a function of time. Suppose there was just enough matter to have an observer and a couple of galaxies. $\rho = 0$ would still be a good approximation, but now we can observe stars and measure their redshift. What in this universe is the relation between proper distance $d_p(t_0)$ and redshift z?

2. In a flat universe with $H_0 = 70$ km s^{-1} Mpc^{-1}, you observe a galaxy at a redshift $z = 7$. What is the current proper distance to the galaxy, $d_p(t_0)$, if the universe contains only radiation.

3. The predicted number of neutrinos in the cosmic neutrino background is $n_\nu = (3/11)n_\gamma = 1.12 \times 10^8 \text{m}^{-1}$ for each of the three species of neutrino. What must be the sum of the neutrino masses, $m(\nu_e) + m(\nu_\nu) + m(\nu_\tau)$, in order for the density of the cosmic neutrino background to be equal to the critical density?

4. (a) Show that

$$\frac{H(t)^2}{H_0^2} = \frac{\rho(t)}{\rho_{c,0}} + \frac{1 - \Omega_0}{a(t)^2}$$

   (This was done in class. If you took good notes, you can just copy them.)

   (b) Consider a curved universe which contains only matter. Show that

$$H_0 t = \int_0^a \frac{da}{[\Omega_0/a + (1 - \Omega_0)]^{1/2}}$$

   (c) Suppose $(1 - \Omega_0)$ is negative. Do the integral and make a plot of a as a function of t.