Cosmology Problem Set #3

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The following exercises are due Friday, February 16.

1. A rocket lifts off from earth and accelerates to some velocity less than the escape velocity. ($v \ll c$) It coasts for a while using no fuel and then falls back to its liftoff point. A comparison is made between the reading on the rocket's clock with that of a clock that stayed on Earth.

   (a) Which of the two rockets followed a geodesic? Which clock counted the smaller amount of time?

   (b) Neglecting the rotation of the Earth, integrate the rocket's proper time between liftoff and the rocket's return to Earth. Use the metric appropriate for a weak static gravitational field. Compare the flight-time measured by the on-board clock with that measured by the Earth-bound clock.

2. Consider a 4-dimensional Euclidean space with coordinates x, y, z, w and the metric $dS^2 = dx^2 + dy^2 + dz^2 + dw^2$. A "3-sphere" of radius a can be defined by the constraint $x^2 + y^2 + z^2 + w^2 = a^2$. Show that the distance dS between nearby points on the surface of the 3-sphere is given by the Robertson-Walker metric with $k = 1$ with $x = ra\sin\theta\cos\phi$, $y = ra\sin\theta\sin\phi$ and $z = ra\cos\theta$.

3. Consider a $k = 1$ universe. Explain why an object at $\chi = \pi$ has an angular size (seen by an observer at $\chi = 0$) of $\Delta\Omega = 4\pi$, i.e. the object covers the whole sky.