Predicting Climate Change
When Data Fails Us

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Graduate Students: 15, Undergraduate Students: 2.
Focus Problems: COMPLEX PROBLEMS

Weather/Climate

Hydrogeology

Computer Vision


UQG
Climate, Mathematics, Predictions

- Abrupt Climate Change
- Mathematics in Climate Research
- The Prediction Problem
Abrupt Climate Change
Global warming is a contemporary fact.

The Earth has undergone many sudden and abrupt changes in its climate.

Estimated $9T in losses for a 3 degree change (Stern Report, UK)
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- Can present global warming trigger an abrupt climate change?
- Are human activities contributing to global warming?
Ice Age: as a Consequence of Global Warming?

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- Can *present* global warming trigger an abrupt climate change?
- Are human activities contributing to global warming?
Global Warming is Real: The Data
Global Warming is Likely

Compounding and Compelling Evidence:

There is no known way to attribute the data trend solely to natural phenomena, and further, correlations in the data cannot be ascribed to chance or known natural causes.
The Earth is experiencing a warming trend.
Global warming is likely to continue.
The scientific consensus is that human activities are implicated.
What Distinguishes Weather from Climate?

A Matter of Scales...

Geophysical Fluid Dynamics

Planetary fluid dynamics, thermodynamics, chemistry

Examples of Weather

- TV forecasts
- Hurricanes
- Flooding
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Climate Examples
- El Niño/La Niña
- Ice Ages
- The Seasons
- The Thermohaline Circulation
Climate: Some Basic Facts

Sun radiation is primary driver
Climate: Some Basic Facts

Equator receives more concentrated solar energy:
Climate: Some Basic Facts

Equator receives more concentrated solar energy:

The area of the sunbeam is spread over 1.15 square meters.

Energy per unit area is 87% of the intensity at the equator.
Climate: Some Basic Facts

Equator receives more concentrated solar energy:

The area of the sunbeam is spread over 2 square meters.

Energy per unit area is 50% of the intensity at the equator.

Images due to Randy Russell, UCAR 2005
Poles would be colder where it not for heat convection and the effects of Earth’s rotation:
Climate: Some Basic Facts

Humidity affects the Atmosphere’s heat capacity and the amount of sun energy reaching the Earth surface:
Climate: Some Basic Facts

The Greenhouse Effect

- Solar radiation passes through the clear atmosphere.
- Some solar radiation is reflected by the atmosphere and Earth's surface.
- Outgoing solar radiation: 103 Watt per m².
- Some of the infrared radiation passes through the atmosphere and is lost in space.
- Net outgoing infrared radiation: 240 Watt per m².
- Some of the infrared radiation is absorbed and re-emitted by the greenhouse gas molecules. The direct effect is the warming of the Earth's surface and the troposphere.
- Surface gains more heat and infrared radiation is emitted again.
- Solar energy is absorbed by the Earth's surface and warms it... 158 Watt per m².
- ...and is converted into heat causing the emission of longwave (infrared) radiation back to the atmosphere.

Sources: Knockhaggen University College, University of Oxford, EPA, IPCC.
Climate: Some Basic Facts

The Greenhouse Effect

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Sources: Oxford University College, University of Oxford, EPA, IPCC.
Climate: Some Basic Facts

The Greenhouse Effect: water vapor and other gases trap heat.
Oceans and Climate

What’s the role of Oceans in Climate?

- Account for 70% of Earth’s surface (all continents can fit in Pacific Ocean).
- Water has 1000 times higher heat capacity than air.
- Largest repository of carbon (93%), when compared to atmosphere and biosphere.

Some papers on the role of the oceans in the carbon cycle:
What’s the role of Oceans in Climate?

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The Thermo (Temperature) Haline (Salt) Circulation (THC)
The sea water density: \( \rho = \rho_0 + \alpha S - \beta T \).

Present heat transport, about 1 PW \((10^{15} W)\) northwards
The Conditions at the Sea Surface

TEMPERATURES

SALINITY

Sea-surface temperature [°C]

Sea-surface salinity [PSU]
THC Predictions, Glacial Times

GLACIAL TIMES

Lower temps and CO$_2$ levels. Transitions triggered by freshwater changes.

- Basic State: Stable
- (H) Heinrich Event: Unstable
- (D/O) Daansgard-Oeschger: Unstable

Image from Potsdam Institute for Climate Impact Research
Ocean's Role in Climate Change

THC Predictions, Present

PRESENT TIMES

Higher temps and CO₂ levels.
- (H) NADW shut-down: Stable
- Cold State: Not Found
- (D/O) present state: Stable
Daansgard-Oeschger Sequence of Events

Source: Ganopolski and Rahmstorf, 2001
How does the THC Shut Down?

One of many shut down sequences:

![Diagram showing the THC shut down sequence]

"there’s observational evidence of a drop in salinity, compared to 1965”, say Scientists at Woods Hole Institute of Oceanography.
Is a THC change happening now?

Greenland ice cover in summer:
What will happen if the THC shut down?
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- Severe decades/centuries-long winters in N. Atlantic, e.g. Lower temperatures in Europe and North America.
- Abrupt regional cooling may occur even as the earth, on average, continues to warm.
- Extreme weather, more intense hurricanes.
What will happen if the THC shut down?

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Mathematics in Climate Research
Complex Modeling..when data is poor

Global Climate Models/General Circulation Models

Weakly three-dimensional "primitive equation formulation"
Linear versus nonlinear Dynamics

A system is linear if it obeys the superposition principle (and scaling):

Otherwise, the system is not linear.
Reduced Modeling

- Can be used to understand some specific aspect of the complex model.
- Bound ranges of model parameters.
- Hypothesize about climate.

Mathematical Analysis

Ed Lorenz (1963) develops a “3-mode” model for convection in the atmosphere.

The model, though fully deterministic, suggests that the atmosphere exhibits quasiperiodic behavior that has seemingly random and abrupt changes.
Mathematical Analysis of the Ocean THC Equations

Climate Equations Have More Than One Climate Solution

Stommel (1961) pioneered this type of analysis on simplified models.

Climate System Questions:
- Are these climates stable?
- What could perturb the state and cause a switch?
- Can human activities be such a perturbation?
Stochastic Parameterizations

When certain processes are essential yet not well understood....

Washington, D.C. 10 April 2009, "NOAA has released its proposal for the use of the $830 million provided to the agency in the stimulus bill... NOAA Climate Computing and Modeling ($170 million): The two HPC sites will...create jobs in manufacturing, construction, and software engineering."
Computational Advances

Moore’s Law: number of integrated circuits doubles, per unit area, about every 18 months\(^1\)

Table: Algorithms, rather than machine speed are responsible for high performance computing.

<table>
<thead>
<tr>
<th>(Y) Years</th>
<th>Moore’s Law</th>
<th>Algorithm G $O(n^3)$</th>
<th>Algorithm CG $O(n)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>$2^2$</td>
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</tr>
<tr>
<td>6</td>
<td>$2^3$</td>
<td>20</td>
<td>8,000</td>
</tr>
<tr>
<td>8</td>
<td>$2^4$</td>
<td>25</td>
<td>16,000</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>Y</td>
<td>$2^{Y/2}$</td>
<td>$33.5 \times 10^6$</td>
<td>$2^{Y/2} \times 1,000$</td>
</tr>
<tr>
<td>50</td>
<td>$\approx 33.5 \times 10^6$</td>
<td>323</td>
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\(^1\)Good for the next 2 decades, maybe...
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Data Access: When data is plentiful

High energy physics research creates the web. Data Mining: Classification, Regression, Association, Sequence, Clustering.

Google’s link search engine ("page ranking") is fundamentally a linear-algebraic strategy on networks.
Sensitivity Analysis

Suppose $y = f(x)$. Want measure how sensitive the outcome $y$ is to changes in the "input" $x$. 

![Graph 1: $y = f(x)$](image1)

![Graph 2: $|dy/dx|$](image2)
Sensitivity Analysis

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![Graphs illustrating sensitivity analysis](image-url)
Sensitivity Analysis

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Could use the derivative $\frac{df}{dx}$:

At $x_0$ the relative change in output $\Delta y/y$ to relative change in input $\Delta x/x$ can be estimated by computing the fair size (a norm) of

$$\left| \frac{\partial f}{\partial x} \right| |x|$$

for $x$ in the neighborhood of $x_0$.

How does one compute a high dimensional gradient without making numerical approximations?

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Sensitivity Analysis

Sensitivity of Drake-passage flow to bottom topography.

- Naive differentiation finite-difference approximation: 23 days.
- Automatic differentiation (ADIFOR): 22 mins.

Some (well-known) Open Research Avenues

- The coupled ocean/atmosphere models are *not consistent*.
- Ocean ice models need to be improved.
- Water cycle models need to incorporate latest developments.
- The convection/humidity in atmospheric models need to be improved.
- Better subgrid parameterizations need to be developed.
- The complex codes need to be sped up.
- DATA, DATA, DATA.
The Prediction Problem
A Major Challenge in Climate Change and Beyond: Variability

What will happen? Are human activities responsible for warming?

- How good are our models and data?
- What are the possible scenarios?
- How large must a perturbation be to cause a significant change?
The Prediction Problem
The Prediction Problem

![Graph showing CO₂ concentration from 1960 to 2020](image-url)
The Prediction Problem
The Prediction Problem

![Graph showing CO₂ concentration from 1960 to 2020]

- CO₂ concentration, ppm
- Year
Open Research Questions

What makes climate science controversial?
Climate Variability and Uncertainty are major technical problems (p.27, 2007 IPCC report)

Sensitivity, Variability, and Uncertainty

- Data with partial information content.
- Poor data coverage.
- Poorly constrained model dynamics and data variability.
- Complex phenomena.
- Now-Predict.

But these are nothing more than ideal conditions for controversy...
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Complicated Complexity

Politically-charged Problems

- Medicine
- Evolution
- Climate

Medicine research: design experimental protocols with which to gather compelling evidence that a hypothesis is made definite
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Medicine research: design experimental protocols with which to gather compelling evidence that a hypothesis is made definite
The Controversies Arise

- Not clear what is meant with the predictions nor how they were obtained.
- Experimental protocols for testing hypothesis are sloppy or obscure.
Predictions

In nonlinear problems there is no single predictor:

Maybe Data Isn’t Failing Us: Data Assimilation

Need to do it right:

- Guarantee statistical convergence of moments.
The Prediction Problem

*When data does *not* fail us...*
The Prediction Problem

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When data fail us...
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The Prediction Problem

*When data fails us...*

\[
dx = 4x(1 - x^2)dt + \kappa dW_t
\]

\[
x(0) = x_0
\]
The Observations
Maybe Data Isn’t Failing Us: Data Assimilation

Combine data and models, constrain the errors....

**but how?**

Bayes Theorem:

\[ P(X|Y) \propto \text{likelihood} \times \text{prior} \]

Some papers on data assimilation:
Maybe Data Isn’t Failing Us: Data Assimilation

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Also: Need to be clear what it is that you are going to estimate:
- Find mean history $X(t)$ conditioned on observations $Y(t_m)$
- Such that $X(t)$ minimizes the (trace) of the Covariance.
Statistically-Convergent Variance-minimizing History and Uncertainty

Figure: KSP Filter

Figure: KSP Smoother
Concluding Remarks

- The Earth is experiencing a warming trend. It is likely correlated to human activities.
  - The scientific consensus is that the anthropogenic (human) component is important.
- There are several climate states, warming can induce a state change.
  - An abrupt climate can occur when a perturbation shifts an equilibrium climate state to another state.
  - Abrupt and severe cooling is one such state.
- The *precautionary principle* should be applied: if an action might cause harm, even if no scientific consensus exists that this is to be the case, the burden of proof falls on those who take this action.
Concluding Remarks (UQG GOALS)

- We need to develop a different approach to using complex systems.
  - Models and data need to be used, even if errors are present.
  - Uncertainty quantification and sensitivity analysis techniques need to be developed.
  - Predictions need to be clear.
  - Hypothesis testing protocols should be designed and used.
  - Statistical convergence would be nice....
Concluding Remarks

- Applicable mathematics research will play an ever increasing role in climate change research:
  - Uncertainty quantification, sensitivity analysis, computing, networks, stochastic parameterizations, modeling.
  - An opportunity in geoscience training arises.
Uncertainty Quantification, the Next Frontier

Further Information

Popular and Technical Science Sources:

On the Thermohaline Circulation and Abrupt Climate Change:
Potsdam Institute for Climate Impact Research http://www.pik-potsdam.de/

On Global Warming:

Juan M. Restrepo
Uncertainty Quantification Group
Mathematics Department, University of Arizona
www.physics.arizona.edu/~restrepo

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