

HW 1-way Wave Equations

(1) Solve the IVP and sketch the characteristics

(a) $u_t + 3u_x = 0 \quad t > 0, x \in \mathbb{R}^1$
 $u(x, 0) = \sin x$

(b) $u_t - x^2 u_x = 0 \quad t > 0, x \in \mathbb{R}^1$
 $u(x, 0) = x + 1$

(2) Find the "shock time t_b " for
 $u_t + u u_x = 0$

$$u(x, 0) = \begin{cases} 1 - x^2 & |x| \leq 1 \\ 0 & |x| > 1 \end{cases}$$

(3) With $a > 0$, show that the solution to
 $u_t + u u_x + a u = 0, t > 0, x \in \mathbb{R}^1$, and $u(x, 0) = f(x)$

is $u = f(\xi) e^{-at}$, $x = \frac{f(\xi)}{a} (1 - e^{-at}) + \xi$.

find a condition on f that guarantees no shocks will form.

4) Find a solution to

$$\begin{cases} 2xu u_x + 2t u u_t = t(u^2 - x^2) - t^3 \\ u(x, 0) = f(x) \end{cases} \quad t > 0, x \in \mathbb{R}^1$$

hint: solve for $u^2(x, t)$