

Wave Eq HW

(1) Find solution to $\frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = \Delta u$ $\begin{matrix} 0 < x < a \\ 0 < y < b \\ t > 0 \end{matrix}$

~~u(x,y,t)~~ B.C. $\begin{cases} u(0,y,t) = u(a,y,t) = 0 \\ u(x,0,t) = u(x,b,t) = 0 \end{cases}$ I.C. $\begin{cases} u(x,y,0) = f(x,y) \\ \frac{\partial u}{\partial t}(x,y,0) = 0 \end{cases}$

where ~~u(x,y,t)~~ $\sin \frac{\pi x}{a} \sin \frac{\pi y}{b} = f(x,y)$

(2) Find solution of $\frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ $\begin{matrix} 0 < x < L \\ t > 0 \end{matrix}$

B.C. $u(0,t) = u(L,t) = 0$

I.C. $u(x,0) = f(x)$ $u_t(x,0) = g(x)$

where $f(x) = \begin{cases} \frac{hx}{x_0} & 0 \leq x \leq x_0 \\ \frac{h(L-x)}{L-x_0} & x_0 \leq x \leq L \end{cases}$ and $g(x) = 0$

(3) Find solution of $\frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + 3 \cos 3t e^{-x}$

$\frac{\partial u}{\partial x}(0,t) = 0$ $u(L,t) = 0$

$\frac{\partial u}{\partial t}(x,0) = 0$ $u(x,0) = \cos \frac{\pi x}{2L}$