HW2 Fourier Series (FS)

① Find (FS) of $f = \sin^2 x \cos x + 3\cos^3 x + 2\sin x \cos x$ w/o computing integrals. Hint: use trigonometric identities.

② The Complex FS for $f(x)$ complex is

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{ik_n x} \qquad k_n = \frac{2\pi n}{p}, \ c_n \text{ is complex.}$$

$2l = p$ is the period

$$c_n = \frac{1}{p}\int_0^p e^{-ik_n x} f(x)\,dx$$

(a) Compute complex FS of e^x $0 \le x \le h$

(b) Confirm that c_0 is the average of e^x over $0 \le x \le h$.

(c) Suppose $f(x)$ is purely real, Show that

(i) for $n \neq 0$: $c_{-n} = \overline{c_n}$ (the negative coefficients must be equal to the complex conjugate of the positive coefficients.

(ii) for $n = 0$ c_0 is real

compare to $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n\pi x}{l}$

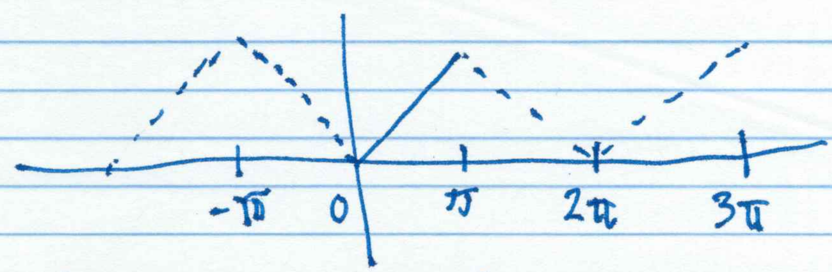
(3) Parseval's Theorem (FS). Informally, show that

$$\frac{1}{2l}\int_{-l}^{l}|f(x)|^2\,dx = a_0^2 + \sum_{n=1}^{\infty}\left(a_n^2+b_n^2\right)$$

hint: form $f \cdot f$ ~~then~~ using series representation for each f. Then integrate both sides over $(-l,l)$, divide by $2l$. Use orthogonality.

(4) Compute the FS for $f(x)=x$, for $0\le x\le\pi$

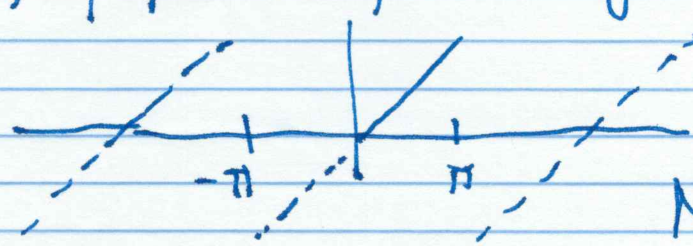
(a) by periodically extending evenly:



$g(x)=|x|$
for $-\pi\le x\le\pi$

Note $f(x)=g(x)$ for $0\le x\le\pi$

(b) by periodically extending oddly:



$h(x)=x$
for $-\pi\le x\le\pi$

Note $f(x)=h(x)$ for $0\le x\le\pi$

(c) At what order do the terms in the series drop in $\frac{1}{n^p}$?

(d) What are the b_n's in (a), What are the a_n's in (b)?