FINAL

(1) Solve

DE $\quad \dfrac{\partial u}{\partial t} - \nu \dfrac{\partial^2 u}{\partial x^2} = x \qquad 0<x<L\,,\ t>0$

BC $\quad \begin{cases} \dfrac{\partial u}{\partial x}(0,t) = 0 & t>0 \\ \dfrac{\partial u}{\partial x}(L,t) = -L^2/2\nu & t>0 \end{cases}$

IC $\quad u(x,0) = \cos\dfrac{3\pi x}{L} - \dfrac{L^2 x}{2\nu} \qquad 0<x<L$

subject to constraint

$$\frac{1}{L}\int_0^L u(x,t)\,dx = 0 \qquad \forall t \geq 0$$

(ii) What is $u(x,t)$ as $t \to \infty$?

(2) Solve
(i) $\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + g(x,t) \qquad 0<x<L,\ 0<t$

B.C. $u(0,t) = u(L,t) = 0$

I.C. $u(x,0) = \frac{\partial u}{\partial t}(x,0) = 0$

(ii) find solution when

$$g(x,t) = \begin{cases} Q(x)/r & 0 \le t < r \\ 0 & r < t \end{cases}$$

(iii) Determine solution when $Q(x) = \sin\frac{\pi x}{L}$

HINT: Solve for $0 \le t < r$, then for $t > r$.

(3) Let D be a rectangle $0<x<a$, $0<y<b$. Find formula for the solution of the Neumann problem for Poisson's Equation in D:

D.E. $\Delta u = -g(x,y)$

B.C. $\frac{\partial u}{\partial n} = 0$ on all edges of D

by assuming a solution of the form

$$u = \sum_{nm} c_{m,n} \varphi_{m,n}(x,y)$$

where $\varphi_{m,n}$ are eigenfunctions of the

Problem D.E. $\Delta\varphi + \lambda\varphi = 0$ in D

B.C. $\dfrac{\partial\varphi}{\partial n} = 0$ on all edges of D

Hint: At some stage in the calculation for u, you'll need to assume

$$\iint_D q \, dx\, dy = 0 \quad //$$

(4) Find a bounded solution of

D.E. $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial x \partial y} + \dfrac{\partial^2 u}{\partial y^2} = 0 \quad \begin{cases} x \in \mathbb{R} \\ 0 < y < \infty \end{cases}$

$u(x,0) = f(x)$

Where $f(x)$ is bounded and absolutely integrable on $\mathbb{R}$.

Express answer in the form $u(x,y) = \displaystyle\int_{-\infty}^{\infty} k(x-\xi, y) f(\xi)\, d\xi$

<u>HINT:</u> $\mathcal{F}^{-1}\left(e^{ikb}\hat{G}(k)\right) = g(x-b)$ and $\mathcal{F}^{-1}\left(e^{-|k|y}\right) = \dfrac{1}{\pi}\dfrac{y}{y^2+x^2}$

(5) Formulate and solve the problem of radial heat flow in a disc of radius R, when the circumference of the disc is insulated and the initial temperature

$$f(r) = \begin{cases} 1 & 0 \le r < R/2 \\ 0 & R/2 \le r < R \end{cases}$$