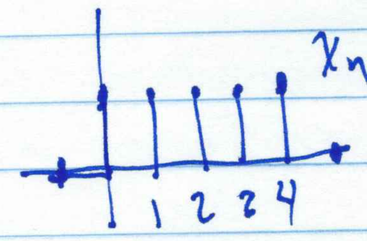
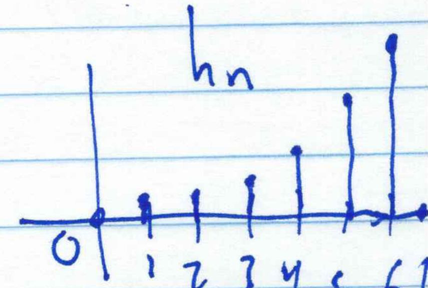
(from S&S Oppenheim & Willsky)

ex) $x_n = \begin{cases} 1 & 0 \le n \le 4 \\ 0 & \text{otherwise} \end{cases}$



$h_n = \begin{cases} \alpha^n & 0 \le n \le 6 \\ 0 & \text{otherwise} \end{cases}$



Compute $y_n = \sum_{j=-\infty}^{\infty} x_j h_{n-j}$

for $n < 0$ $y_n = 0$

for $0 \le n \le 4$

$$y_n = \sum x_j h_{n-j}$$

$$= \begin{cases} \alpha^{n-j} & 0 \le j \le n \\ 0 & \text{otherwise} \end{cases}$$

$\therefore$ $y_n = \sum_{j=0}^{n} \alpha^{n-j}$. let $r = n-j$

$$= \sum_{r=0}^{n} \alpha^r = \frac{1-\alpha^{n+1}}{1-\alpha}$$

for $n>4$ but $n-6\leq 0$ (i.e. $4<n\leq 6$).

$$x_j h_{n-j} = \begin{cases} \alpha^{n-j} & 0\leq j\leq 4 \\ 0 & \text{otherwise} \end{cases}$$

$$\therefore y_n = \sum_{j=0}^{4} \alpha^{n-j} = \alpha^n \sum_{j=0}^{4} \alpha^{-j} = \alpha^n \frac{1-\alpha^{-5}}{1-\alpha^{-1}} = \frac{\alpha^{n-4}-\alpha^{n+1}}{1-\alpha}$$

for $n>6$ but $n-6\leq 4$ (i.e. $6<n\leq 10$)

$$x_j h_{n-j} = \begin{cases} \alpha^{n-j} & (n-6)\leq j\leq 4 \\ 0 & \text{otherwise} \end{cases}$$

$$y_n = \sum_{j=n-6}^{4} \alpha^{n-j} \qquad \text{let } r = j-n+6$$

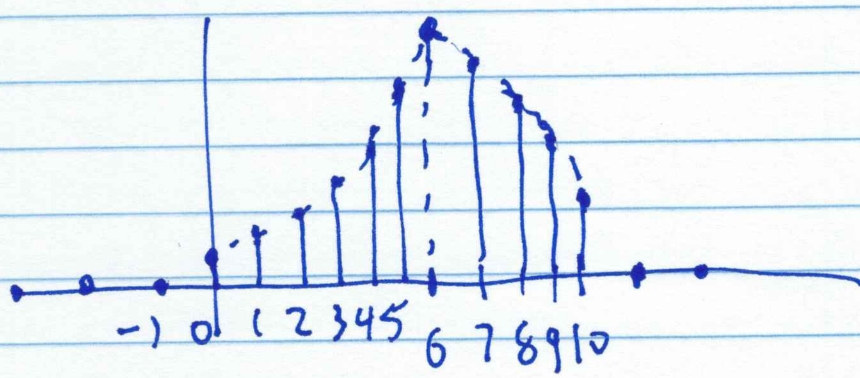
$$= \sum_{r=0}^{10-n} \alpha^{6-r} = \alpha^6 \sum_{r=0}^{10-n} (\alpha^{-1})^r = \alpha^6 \frac{1-\alpha^{n-11}}{1-\alpha^{-1}}$$

for $(n-6)<4$ or $n>4$

$$y_n = 0$$

So

$$y_n = \begin{cases} 0 & n < 0 \\ \dfrac{1-\alpha^{n+1}}{1-\alpha} & 0 \le n \le 4 \\ \dfrac{\alpha^{n-4} - \alpha^{n+1}}{1-\alpha} & 4 < n \le 6 \\ \dfrac{\alpha^{n-4} - \alpha^{7}}{1-\alpha} & 6 < n \le 10 \\ 0 & 10 < n \end{cases}$$



//