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G.I. Taylor and the Trinity test

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The story is often told of the calculation by G.I. Taylor of the yield of the first ever atomic bomb exploded in New Mexico in 1945. It has indeed become a staple of the classroom whenever dimensional analysis is taught. However, while it is true that Taylor succeeded in calculating this figure at a time when it was still classified, most versions of the story are quite inaccurate historically. The reality is more complex than the usual accounts have it. This article sets out to disentangle fact from fiction.

Keywords: dimensional analysis; point source solution; atomic bomb; 'Trinity' test

AMS Subject Classifications: 00A73; 76L05; 01A60

1. Introduction

One of the staple examples used in the teaching of dimensional analysis is the story of the way in which the yield of the first atomic explosion (the 'Trinity test') was calculated from declassified film at a time when the yield itself was still secret. See, e.g. [1,2]. What is normally advanced in such discussions, however, is a simplified version of the approach actually used; indeed one which necessarily remains incomplete. The incompleteness is covered by a number of subterfuges, most of them in fact wrong.

In what follows, I will present first the (now standard) simplified, or rather oversimplified, approach, and will proceed to a discussion of its limitations in the light of what actually happened.

2. The simplified approach

If we apply the standard techniques of dimensional analysis to the problem of a powerful explosion and its effects, we begin by listing the physical variables likely to be involved. The explosion is modelled by idealizing its genesis as originating in a point and so giving rise to a spherically symmetric shock wave. Interest focuses on the wave-front itself, seen as a boundary between external (normal) air on one side and superheated, highly compressed air inside a fireball on the other. If we then list the physical quantities likely to be involved, we find:

E, the energy released by the explosion; *R*, the radius of the fireball; *t*, the time since detonation; *p*, the pressure inside the fireball; p_0 , the pressure outside the

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fireball, i.e. that of normal air; ρ , the density of the air inside the fireball, and ρ_0 , the density of (ordinary) air outside the fireball.

According to the Buckingham Pi Theorem [1,2], these seven physical quantities may be combined into four independent dimensionless ratios, that is to say, quantities independent of the units of measurement employed. This may be done in a number of ways, but the following is convenient:

$$\Pi_1 = R \left[\frac{Et^2}{\rho_0} \right]^{-1/5}, \quad \Pi_2 = \frac{p_0^5 t^6}{E^2 \rho_0^{-3}}, \quad \Pi_3 = \frac{\rho}{\rho_0}, \quad \Pi_4 = \frac{p_0}{p}$$

The required law is then (again by the theorem)

$$F(\Pi_1, \Pi_2, \Pi_3, \Pi_4) = 0.$$

or equivalently,

$$\Pi_1 = f(\Pi_2, \Pi_3, \Pi_4).$$

But now, Π_3 is the ratio of the density of very light superheated air to the density of ordinary air, and so is nearly zero; likewise Π_4 is the ratio of the pressure of ordinary air to that inside an exploding fireball, and so it is also very nearly zero. It takes a little more thought to see that $\Pi_2 \approx 0$, but we may reflect that this ratio compares a combination of quite ordinary quantities to the square of the energy released by an atomic blast, and so it is also very small. The result of this further analysis is to give us the equation:

$$\Pi_1 = f(0, 0, 0) = \text{constant}$$

In other words,

$$E = K\rho_0 R^5 t^{-2},$$
 (1)

where K is a dimensionless constant, in short a simple number.

Let us call Equation (1) the *basic equation*. It may be cast in many equivalent forms; in particular:

$$R = \left[\frac{Et^2}{K\rho_0}\right]^{1/5} = \left[\frac{E}{K\rho_0}\right]^{1/5} t^{2/5} = at^{2/5} \quad \text{say.}$$
(2)

Here, a is a constant, although not a dimensionless one.

3. Completing the calculation

The urban myth is that this simplified approach was used by Taylor to calculate the value of E for the 'Trinity' test, the world's first ever atomic explosion. Barenblatt [1] gives a (marginally different) account of the simplified approach, and then says: 'The discussion presented above is due to G. I. Taylor, who processed data from a movie film of the expansion of a fireball taken during an American nuclear test...'. Bluman [2] is a little more cautious: '[the simplified account] is a dimensional analysis viewpoint of Taylor's astounding deduction'.

If the aim is to calculate the value of E, then we apply Equation (2) to (in theory, a single) photograph giving a pair of values for R, t and so deduce the value of a.

Then, because ρ_0 is known, *E* may be calculated if only we know the value of the dimensionless constant *K*. This is as far as dimensional analysis alone can take us.

So the question arises as to how to proceed from here. Barenblatt states correctly that the determination of K is a problem in gas dynamics, and (also correctly) attributes its solution to Sedov [3,4], Taylor [5,6], and von Neumann [7]. Bluman, however, says: 'Light explosives experiments can be used to determine F(0) [i.e. K^{-5} in my notation]. It turns out that [its value] is approximately 1'. This is misleading, as I will demonstrate in Section 11, although it is an easy trap to fall into, and in fact I plead guilty of such a lapse in some of my own presentations of the problem. In many applications of dimensional analysis, such an approach is appropriate. However, this is not the case here.

The reader is invited to explore what is available on the internet. Google search the term Dimensional Analysis Atom Bomb. One will find examples of the two approaches just outlined, along with others: 'let's assume the constant is approximately 1', and, as if this were not enough, at least one website has it fall into a 'black hole' between two consecutive Microsoft PowerPoint[®] slides, and so sets it equal to 1 by default! In order to protect the guilty, I forbear from giving details!

4. What Taylor actually did

Taylor's own account of the matter is available [5,6], as is an authoritative retelling of the actual story by Barenblatt [8], who recounted the case in the course of his inaugural lecture on his elevation to the G. I. Taylor professorship in Fluid Mechanics at the University of Cambridge.

Taylor [5] states that he was approached in 1941 by the UK Ministry of Home Security and told that 'it might be possible to produce a bomb in which a very large amount of energy would be released by nuclear fission'. His task was to report on the likely effect of such a weapon.

Central to Taylor's analysis was the approximation of the initial detonating device by a mathematical point; equally important was the assumption of spherical symmetry. Now the situation inside the fireball involves the density ρ and the pressure p (as introduced in Section 2 above) and also the outward velocity u. All these quantities are seen as dependent on a radial co-ordinate r and the time t since detonation.

He thus set up three simultaneous partial differential equations: an equation of motion

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} = -\frac{1}{\rho} \frac{\partial p}{\partial r}$$
(3)

an equation of continuity

$$\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right) = 0 \tag{4}$$

and an equation of state

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}\right)(p\rho^{-\gamma}) = 0, \tag{5}$$

where γ is the adiabatic constant. (It is here assumed that the air inside the fireball is not exchanging energy with that outside.)

He then sought a similarity solution for these equations. To this end, he assumed that

$$\frac{\mathrm{d}R}{\mathrm{d}t} = AR^{-3/2} \tag{6}$$

where R was the radius of the fireball (as in Section 2 above) and A was a constant (but not a dimensionless one). He thus reduced the three partial differential equations to three ordinary ones:

$$\phi'(\eta - \phi) = \frac{1}{\gamma} \frac{f'}{\psi} - \frac{3\phi}{2} \tag{7}$$

$$\frac{\psi}{\psi} = \frac{\phi' + 2\phi/\eta}{\eta - \phi} \tag{8}$$

$$3f + \eta f' + \frac{\gamma \psi'}{\psi} (\varphi - \eta) f - \varphi' f' = 0,$$
(9)

where φ , ψ , f, and η are scaled measures of radial velocity, density, pressure, and radial distance, respectively, the prime indicates differentiation with respect to η , and all of these quantities are now in dimensionless form.

It should be noted that, although a similarity solution is sought (via Equation (6) and aspects of the scaling) and although the new form of the basic equations is dimensionless and although these changes to the equations are indeed related to the considerations underlying the simplified approach, we *do not* see here a straightforward use of dimensional analysis (for an excellent account of the connections between such considerations and dimensional analysis itself, see the text by Bluman and Cole [9]).

Equations (7)–(9) present as a well-defined system with η as the sole independent variable and γ the only parameter. They are first order and so require three boundary conditions. These Taylor took from the Rankine–Hugoniot relations for shock-waves. Under the condition which I have written as $\Pi_4 \approx 0$, these reduce to:

$$\phi(1) = \frac{2}{\gamma + 1}, \quad \psi(1) = \frac{\gamma + 1}{\gamma - 1}, \quad f(1) = \frac{\gamma + 1}{2\gamma}.$$
 (10)

At this stage of the calculation, he paused to consider the energy, and found it could be expressed as

$$E = B(\gamma)\rho_0 A^2, \tag{11}$$

where $B(\gamma)$ was expressed as a definite integral involving φ , ψ , and f. The remaining work was the determination of these functions.

To this end, he employed two separate approaches. The first was a step-by-step integration from the value $\eta = 1$, the second a search for tractable approximate forms for φ , ψ , and f. The completion of the calculation by either of these means needed the value of γ .

5. The value of γ

Ordinary air is composed almost entirely of oxygen and nitrogen, both diatomic gases. For a diatomic gas, the value of γ is 7/5, i.e. 1.4. The question arose however as to whether this value would obtain inside the fireball. It could, for example, be that the molecules of oxygen and nitrogen could become dissociated and thus the gas become monotonic with a value $\gamma = 5/3 = 1.666...$

Taylor carried out calculations for both these values, but paid more attention to the case $\gamma = 1.4$. He thus derived the result B(1.4) = 5.36. This cleared the way to a full account of the matter.

6. Parallel developments

At almost exactly the same time as Taylor's involvement with the problem, two other researchers were also considering it. These were John von Neumann in the USA and Leonid Sedov in the Soviet Union. Sedov's work will be discussed in Section 7 below. Here, we examine von Neumann's [7] work.

John von Neumann was involved in the Manhattan Project that resulted in the development of the atom bomb, and in the course of his involvement considered the same questions that had been put to Taylor. His analysis proceeded along broadly similar lines, but in a somewhat simpler fashion. Earlier he established Equation (2), which defines the parameter a. While Taylor had used an Eulerian formulation, von Neumann employed a Lagrangian one. He related the two to each other by means of the equation

$$X = at^{2/5}F(z)$$
, where $z = \frac{x}{at^{2/5}}$ (12)

Here, X is the Eulerian co-ordinate (Taylor's r), x its Lagrangian counterpart, and F a function to be determined. Equation (12) already incorporates the similarity assumption, and so partial differential equations are bypassed.

The other difference between his approach and Taylor's was his replacement of the equation of motion by an energy integral. He was able to show that the energy contained within each sphere z = constant was a function of z alone, which meant that 'the energy flowing into it with the new material that enters is exactly compensated by the work which its original surface does by expanding against the surrounding pressure'. The detailed elaboration of this point led to a single first-order ordinary differential equation (albeit an extremely complicated one!) for the function F(z).

Much of the rest of von Neumann's analysis is concerned with the solution of this equation, but by means of a number of truly ingenious manoeuvres he was able to give an explicit formula (an extremely complicated one) for F(z). This in turn led to an explicit formula for the initiating energy $E(E_0 \text{ in his notation})$ in the form of a Stieltjes integral.

7. Sedov's work

Rather less is known of the genesis of Sedov's interest in the problem [3,4]. It is a plausible speculation that it proceeded from the same military concerns as did Taylor's and von Neumann's, although Sedov nowhere adverts to this. He does draw

attention to related problems of supersonic flight and also considers plane and cylindrical shock-waves as well as spherical. The details of his analysis are, however, broadly similar to those of von Neumann's rather than of Taylor's.

He organized the calculation somewhat differently, using Equation (1) with K=1 to compute a nominal energy E. Then he sought this to relate to the initiating energy E_0 by means of an equation

$$E_0 = \alpha(\gamma)E. \tag{13}$$

He was able to express $\alpha(\gamma)$ as a sum of two definite integrals, and evaluated it explicitly in the case $\gamma = 1.4$.

The model used by all three researchers is sometimes given a name bracketing all three of them, but more usually is known by von Neumann's term *the point source model*.

8. Some comments

Before moving on, let us pause briefly to look at the various contributions by the three researchers. Taylor's analysis was actually the least successful in that, where von Neumann and Sedov found exact solutions, he had only approximate ones.

All three of them gave considerable time and space to the question of the appropriate value to assign to γ , and all three made calculations for values other than 1.4, although they all ended up placing most emphasis on this particular case.

Considering the central role played by γ , and the fact that it is a nondimensionless quantity, it is surprising that the simplified approach ignores it. Were it included in the analysis, then Equation (1), the basic equation, would be slightly altered to

$$E = K(\gamma)\rho_0 R^5 t^{-2} \tag{14}$$

a form given explicitly by Taylor and von Neumann and implicitly by Sedov.

At the time of Taylor's initial involvement in the problem, he gave Equation (14) in a somewhat different form. He had *R* as the subject of the formula. (The notation is also a little different, $K(\gamma)$ is then $S(\gamma)^{-5}$; there is also a minor misprint, ρ for ρ_0 .) There is a reason for writing the equation in this form. The equation (the first in the entire paper) is expected to calculate the extent of the devastation caused by the blast. The somewhat surprising conclusion is that 'an atomic bomb would be only half as efficient, as a blast-producer, as a high explosive releasing the same amount of energy'. This is because much more of the energy is dissipated as heat (and, of course, high explosives *do not* release the same amount of energy).

9. The sequel: Taylor's second paper

Taylor's report was handed to the Ministry on Friday, 27th June, 1941; von Neumann's was submitted only 3 days later (He took the weekend to check the calculations; there are 168 of them, all horrendous!). Both reports were written under the secrecy conditions prevailing in wartime. We may perhaps presume that the same was true of Sedov's. However, Sedov published, albeit in Russian, in the open

literature in 1946 [3]. Taylor was finally cleared to publish in 1949, and his analysis [5] appeared in 1950. The original report was reprinted with only minor changes, the most significant being the inclusion of an appendix updating earlier work on conventional explosives. There it formed the first paper of a pair, and was subtitled 'Theoretical Discussion'. The second [6] had the subtitle 'The Atomic Explosion of 1945'. von Neumann's analysis became a chapter in a Los Alamos research report, which presumably remained classified for some time. It did not find a place in the mainstream literature until after his death [7].

But now let us look at the second part of Taylor's analysis. In 1947, a film of the Trinity test was released by the US Atomic Energy Commission along with a number of still photographs. All in all, there were 25 pictures, each accompanied by an exact time and a length-scale. They were widely circulated and appeared in e.g. *Life* Magazine and elsewhere. Taylor received a compete set and used this to analyse the growth of the fireball. The primary purpose was to test the underlying model, the point source model. It also allowed the test of one aspect of Equation (2). That equation could be rendered invalid if the actual value of γ changed with time.

The 25 photographs gave Taylor 25 paired values of R, t. If we put Equation (2) into logarithmic form, as Taylor did, we find

$$\frac{5}{2}\log_{10} R = \log_{10} t + \frac{5}{2}\log_{10} a, \tag{15}$$

so that a plot of $\frac{5}{2}\log_{10} R$ against $\log_{10} t$ should result in a straight line of slope 1.

The dataset covered the times from 0.1 to 62 ms, really a very short period of time indeed. If even shorter times are considered, then the approximation of the initial charge by a mathematical point will not be valid; for longer times, other phenomena become important (e.g. buoyancy effects leading to the signature 'mushroom cloud'). Barenblatt [10] adduces this as an instance of his 'intermediate asymptotics' and gives precise bounds on the times over which Equation (15) would hold. However, when Taylor plotted the values in this way, he saw an almost perfect fit. Visually, all but the very first line up as predicted.

Actually, the slope of the line of best fit through the remaining 24 is 0.976, but we may take this to be exactly 1 within the limits of measurement of R. If we now seek a value of $\frac{5}{2}\log_{10} R$ that provides a best fit under the constraint that the slope is 1, we discover

$$\frac{5}{2}\log_{10}a = 6.916.$$

(I am not here following Taylor's convention of adding 5 to each side of Equation (15), but in what follows, I *do* follow him in taking the decimal part as 0.915, not 0.916.). This gives a = 583.4, from which we deduce that, in SI units,

$$R^5 t^{-2} = 6.76 \times 10^{13}. \tag{16}$$

(There is actually a minor error in Taylor's paper at this point where he should have 6.76, he has transposed two digits to reach 6.67. This affects the subsequent results in a minor way. As far as I have been able to see, no one else has mentioned this. Either they did not check his work or else they were too polite to point out the error!) The linear fit derived from the photographic data triumphantly vindicated the point source model and the mathematics that flowed from it, but now Taylor was in a position to estimate the value of *E*. Setting $\rho_0 = 1.25$, we find

$$E = 8.45 \times 10^{13} K(\gamma)$$

If we set $\gamma = 1.4$, various values of *K* have been given. Taylor himself gave 0.856, which may be deduced from the value of *B*(1.4) listed in his first paper. von Neumann, using his Stieltjes integral, calculated 0.8510, which is more precise. (My erstwhile colleague, Barrie Milne, has repeated von Neumann's calculation [11] to find 0.85107. Sedov [4] has $\alpha(1.4) = 1.175$, which corresponds to K(1.4) = 0.85106.) Taking the best of these values tells us that $E = 7.19 \times 10^{13}$ J, i.e. 16.9 kilotonnes of TNT. Taylor's estimate was 16.8 kilotonnes [although he should have found 17.5; the error caused by the digit transposition is here almost cancelled out by the improvement in the value of K(1.4)].

It is undoubtedly this aspect of Taylor's second paper that has earned him his fame for the analysis. It also led to him, in Batchelor's words [12], being 'mildly admonished by the US Army for publishing his deductions from their (unclassified) photographs'.

10. How good was the estimate?

Strangely, it is not an easy matter to discover quite what the yield of the Trinity test actually was. The Wikipedia article, *Nuclear weapon yield* [13], actually contradicted itself (at the time of my access) and gave both 19 and 20 kilotonnes for this figure (and there was also a lot of further misinformation to be found there!). According to Batchelor [12], President Truman announced a figure of 20 kilotonnes (presumably once the figure was declassified). General Groves, the military commander of the Manhattan Project, reported a figure of 15–20 kilotonnes [14]. Other values may also be found in the literature.

One puzzling aspect of the whole story is the suggestion that the value assigned to $K(\gamma)$ is too low. Because the figures derived by von Neumann (confirmed by Milne) and Sedov agree so well and correspond quite closely with Taylor's approximate value, we may dismiss any idea of computational error. However, in Section 3 above, I have reported a consensus that the value is very nearly 1. In his inaugural lecture [8], Barenblatt goes further and has: 'he showed that the constant *C* [i.e. our *K*] is close to unity: for $\gamma = 1.4$, C = 1.033'. Barenblatt took this figure, in their notation $\xi_1(1.4)$, from a text by Zel'dovich and Raizer [15, p. 99] and influentially promulgated it in his inaugural lecture [8], generously if erroneously, attributing the figure to Taylor. Zel'dovich and Raizer nowhere say how their figure was calculated, but Barenblatt [16] speculates that it was reached by back-calculation from a presumed yield of 20.3 kilotonnes (or 21.1 kilotonnes if we correct the transpositional error).

It should also be realized that the estimate of the yield is necessarily an *underestimate*. It ignores that portion of the energy that is converted into radiation. Taylor [6], Batchelor [12], and Sedov [4] all advert to this, and Taylor has this to say: 'the effect of radiation... cannot be estimated'. (There is no record of the length of time over which the fireball continued to radiate.)

However, it seems likely that the 1.033 figure is the source of the widespread belief that 'the value is very nearly 1'.

Batchelor is surely right when he finds public opinion correct in giving especial credit to Taylor (over von Neumann and Sedov, although their analyses were in fact better). It was Taylor and Taylor alone who thought to test the validity of the point source solution, and who produced the triumphant logarithmic plot that so thoroughly vindicated it.

11. Conventional explosives revisited

The widespread belief that the value of K was determined by experimentation with conventional explosives needs further discussion. Taylor [5] devoted some time to the discussion of conventional explosives and it was this matter that occupied his attention in the supplement to that first paper. However, the thrust of those discussions was to highlight the difficulty of exact comparison. One point is that conventional explosives need to take up so much space that the point source approximation can become invalid. (1 tonne of TNT occupies a space larger than an 80 cm cube!) Bluman [2] estimates that the explosion of 50 lb (11.5 kg) of TNT would need to be followed for a time less than 0.8 ms while its fireball achieved a radius of 2.2 m. It may not be possible to achieve this level of experimental accuracy.

Furthermore, Taylor's comparisons showed that such extrapolations were far from exact. His principal point was that in an atomic explosion, a greater proportion of the energy was 'wasted', in the sense that it was not available to propagate the blast wave. (Furthermore, the proportion of the energy converted into radiation may also be quite different.) There was certainly no suggestion that data derived from conventional explosives could be scaled to give the value of $K(\gamma)$.

12. Notes on the sources

I have taken the 'simplified approach' from Bluman [2] and Barenblatt [1] in preference to other such accounts, because of the manifest authority of these authors. In contrast, I have ignored the (many) such versions available on the internet. It will be clear from my text above that much of this material is of poor quality. (Indeed, in general, I have avoided giving internet references, in part because of the ephemeral nature of many of them. Readers may, however, seek such material for themselves.) It is also abundantly clear that many of the authors who have referenced Taylor's papers [5,6] have not in fact read them, or else perhaps have sought somehow to improve on them.

Von Neumann's paper was first published as Chapter 2 of *Blast Wave* published as Los Alamos Sci. Lab. Tech. Series, Vol. 7, Part 2, H. Bethe, ed., August 13, 1947, LA-2000. It is unclear to me when it was declassified. In my text, I have relied entirely on the version in his *Collected Works*. This clearly carries a number of improvements on his original; these may or may not have been included in the final version of the Los Alamos technical report.

Sedov's paper [3] was published in Russian, and a purported English translation (Pergamon 1223) is no more than a brief summary. However, the material is, in large

measure, reproduced in his book [4]. This latter account is the very best as a thorough description of how the computations can actually proceed.

Another author, G. J. Kynch, has claimed part of the credit for the point source solution [17], which he referred to as 'a solution due to Taylor, Neumann and Kynch based on a similarity hypothesis for the flow...'. I have not given space to Kynch in the main part of the article. His contribution was to recast von Neumann's exact solution in a somewhat simpler form. At least in the version that saw mainstream publication, von Neumann's paper [7] incorporates Kynch's improvement with due acknowledgement. This is one of the improvements referred to above. Kynch seems to have been unaware of Sedov's work.

Batchelor [12] mentions three further 'discoveries' of the point source solution. One is Sedov's [3]. The other two are not in fact discoveries; both refer to Taylor. However, one of them [18] constitutes an independent discovery of the exact solution found by von Neumann and Sedov. Like their versions, it employs an energy integral.

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