

MATH 342 FINAL EXAM, PART II

NAME:

Instructions:

- You must include all the steps in your derivations/answers. Reduce answers as much as possible, but use *exact arithmetic*. Write neatly.
 - The exam consists of the enclosed *take-home* portion, worth 60%, which you will hand in with the *in-class* portion, worth 40%.
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1. (20 %) (include details of the calculation) Find the singular value decomposition of

(a) $A = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$

(b) $B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(c) $C = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

2. (20 %) Let A consist of the columns $a_1 = [1 \ 2 \ 2]^T$ and $a_2 = [1 \ 3 \ 1]^T$.

(a) Perform a Gram-Schmidt orthogonalization of the matrix A , to obtain $A = QR$. Find Q and R . Show details.

(b) If $A \in \mathbb{R}^{m \times n}$, what are the dimensions of the matrices Q and R ?

3. (20 %) Let

$$A = \begin{bmatrix} 0.2 & 0.4 & 0.3 \\ 0.4 & 0.2 & 0.3 \\ 0.4 & 0.4 & 0.4 \end{bmatrix},$$

let

$$u_0 = \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}.$$

and

$$u_{n+1} = Au_n, \quad n = 0, 1, 2, \dots$$

- (a) Find the eigenvalues of A .
- (b) Diagonalize A .
- (c) Find u_{10} .
- (d) Find $\lim_{n \rightarrow \infty} u_n$