

HW 6

① For $A \in \mathbb{F}^{m \times n}$. Show that A^*A has the same nullspace ^{as} A .

② Let $A \in \mathbb{F}^{m \times n}$ $x \in \mathbb{F}^n$ $b \in \mathbb{F}^m$
such that $Ax = b$

$$\text{let } b = Pb + r \quad r \perp \text{col}(A)$$

where $P = A(A^*A)^{-1}A^*$ is the projection matrix

(a) Show that $P^2 = P$

(b) Show that $P^* = P$

(c) Conversely, show that any symmetric matrix with the property $P^2 = P$ represents a projection.

③ (a) Find the projection of $\underline{b} = [1 \ 2 \ 7]^T$ into the column space of $A = \begin{bmatrix} 1 & -1 \\ 1 & 4 \\ -2 & 4 \end{bmatrix}$. Split $\underline{b} = \underline{p} + \underline{q}$

where $\underline{p} \in \text{col}(A)$ and $\underline{q} \perp \text{col}(A)$

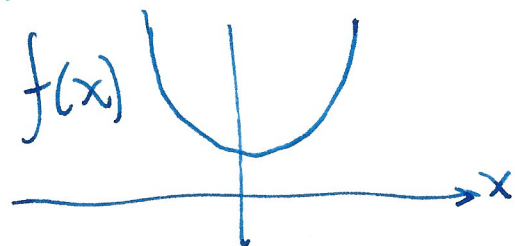
(b) To which subspace of $\mathbb{F}^3$ does $\underline{q}$ belong to?

④ Suppose $S \in \mathcal{L}(W, V)$, $T \in \mathcal{L}(V, W)$ are selfadjoint. Prove that ST is selfadjoint if and only if $ST = TS$ (they commute).

⑤ Find best straight line fit to $(-2, -4)$, $(-1, -3)$, $(0, 1)$, $(2, 0)$

⑥ Fit a plane $z = a + bx + cy$ to $(1, 1, 3)$, $(0, 3, 6)$, $(2, 1, 5)$, $(0, 0, 0)$ in some least square sense. How good is the fit?

⑦ Start with $f(x) = \|Ax - b\|^2$. Differentiate $f(x)$ with respect to x , set to 0 and find the



minimizer, $\hat{x}$. In the figure, what is $A\hat{x}$?