HW3

Download Experiments ~~.~~ Moler, do exercises 5.1c, 5.2 d,e,f, 5.3, 5.4, 5.7

Moler Ch5: 5.1c : note that $\begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix}$ has 2 columns that are LD: $\begin{pmatrix} 2 \\ 6 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

and $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ is ~~also linearly dependent to~~ $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ not in column space of $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$. No solution.

5.2(d) solution $(1,1,1)^T \cdot 0.0\overline{66}$

(e) if you solve by $A\backslash b = [0.2, -1.4, 1.2]^T$. If you use $inv(A) \cdot b = [0,-1,1]^T$. The matrix A is very ill-conditioned

Note that $rref\begin{pmatrix} A \end{pmatrix} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$ ∴ there would be inf # of solutions.

(f) no solution

5.3 Find $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \underset{\sim}{x} = \emptyset \xrightarrow{rref} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \underset{\sim}{x} =$ $\begin{matrix} x_1 - x_3 = 0 \\ x_2 + 2x_3 = 0 \end{matrix}$

$x_1 = x_3 \quad x_2 = -2x_3 \qquad \underset{\sim}{x} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} x_3$

5.4 The Matrix X has the columns from the solution of $A^{-1}\begin{pmatrix} 360 \\ 0 \\ 0 \end{pmatrix}$, $A^{-1}\begin{pmatrix} 0 \\ 360 \\ 0 \end{pmatrix}$, $A^{-1}\begin{pmatrix} 0 \\ 0 \\ 360 \end{pmatrix}$

5.7 $Ri = b$ and $Gv = c$ Verify that the following holds: So $v = Ri \Rightarrow GRi = c$

$\Rightarrow b = G^{-1}c$

3.8 SADW Show that if $A\underline{x} = \emptyset$ has a unique solution then A is left invertible.

So $A^{-1}Ax = I\underset{\sim}{x} = \emptyset$ so $\underset{\sim}{x} = 0$

so the unique solution is $\underline{x} = \emptyset$. If $A = LU$ and has full pivots. ~~they~~ Now, suppose we use the Gauss Jordan procedure to find the n pivots. Then

$E_n \cdots E_2 E_1 A = I$, where each E_i represents the operations required to obtain each pivot. So E_i are matrices.

$E_n \cdots E_2 E_1 = A^{-1}$ can only be the above. Hence, A is left invertible.

6.1 (a) False. Can have no solution

(b) False. Can have inf or no solutions

(c) $Ax = 0$ True, it always has $x = 0$

(d) False. It can have no solution if $Ax = b$, b is not in $Col(A)$

(e) False.

(f) $Ax = 0$ can have $\underset{\sim}{x} = 0$ as solution, or many, but it does not imply that $Ax = b$ has a solution. false

(g) $Ax = 0$ with A invertible has only $x = 0$ as solution

(h) $Ax = b$ where $A \in \mathbb{R}^{m \times n}$ $x \in \mathbb{R}^n$, $b \in \mathbb{R}^m$, the cols of A are in the subspace $\mathbb{R}^m$, the cols have $\dim = r \leq$

(i) $A\underset{\sim}{x} = 0$ where $A \in \mathbb{R}^{m \times n}$, the cols of A are in the subspace of $\mathbb{R}^m$, $\underset{\sim}{x}$ belongs to the Null space (A) and it is a subspace of $\mathbb{R}^n$. Yes

Question No.

Candidate No.

① $\begin{cases} (V_1 - V_2) g_{12} + (V_1 - V_3) g_{13} + (V_1 - V_4) g_{14} = 0 \\ i_1 - i_2 \quad + i_2 - i_3 \quad + i_3 - i_1 \quad = 0 \end{cases}$

⑤ $\begin{cases} (V_5 - V_4) g_{45} + V_5 g_{35} + (V_5 - V_2) g_{25} = 0 \\ i_4 - i_1 \quad i_4 \quad i_1 \quad = 0 \end{cases}$

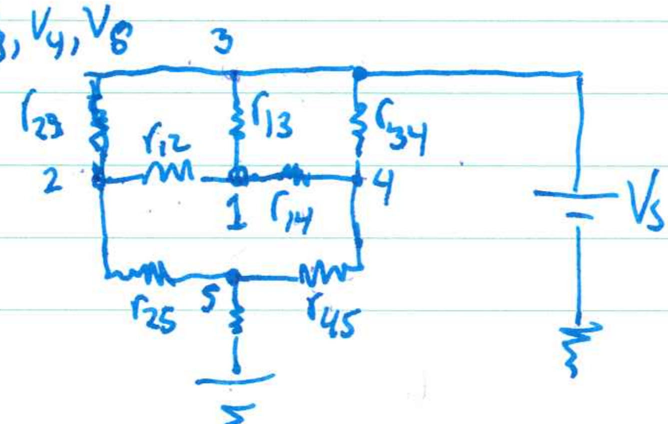
② $\begin{cases} (V_5 - V_2) g_{25} + (V_2 - V_3) g_{23} + (V_1 - V_2) g_{12} = 0 \\ -i_1 \quad + i_2 \quad i_1 - i_2 \quad = 0 \end{cases}$

③ $\begin{cases} (V_2 - V_3) g_{23} + (V_1 - V_3) g_{13} + (V_3 - V_4) g_{34} + g_{35} V_5 = 0 \\ i_2 \quad + i_3 - i_2 \quad - i_3 \quad - i_4 \end{cases}$

④ $\begin{cases} (V_5 - V_4) g_{45} + (V_1 - V_4) g_{14} + (V_3 - V_4) g_{34} = 0 \\ i_4 - i_1 \quad + i_1 - i_3 \quad + i_3 - i_4 \quad = 0 \end{cases}$

Solve for V_5 in terms of V_1, V_2, V_3, V_4, V_S

then get system



(5) Find 2x3 system (2 eqs, 3 unknowns) such that its general solution is

$$x = \begin{pmatrix}1\\1\\0\end{pmatrix} + s\begin{pmatrix}1\\2\\1\end{pmatrix} \quad \text{where } s \in \mathbb{R}.$$

We're looking at $Ax = b$ with $x = x_h + x_p$

$$x_p = \begin{pmatrix}1\\1\\0\end{pmatrix} \text{ and } x_h = s\begin{pmatrix}1\\2\\1\end{pmatrix}$$

hence (A) $\begin{cases} a_{11}x_1^h + a_{12}x_2^h + a_{13}x_3^h = 0 \\ a_{21}x_1^h + a_{21}x_2^h + a_{23}x_3^h = 0 \end{cases}$ OR $\begin{cases} a_{11} + 2a_{12} + a_{13} = 0 \\ a_{21} + 2a_{22} + a_{23} = 0 \end{cases}$

(B) $\begin{cases} a_{11}x_1^p + a_{12}x_2^p + a_{13}x_3^p = b_1 \\ a_{21}x_1^p + a_{22}x_2^p + a_{23}x_3^p = b_2 \end{cases}$ OR $\begin{cases} a_{11} + a_{12} = b_1 \\ a_{21} + a_{22} = b_2 \end{cases}$

by inspection in (A) let $a_{11} = 1$ $a_{12} = 1$ $a_{13} = -3$

$a_{21} = 1$ $a_{22} = -1$ $a_{23} = 1$

satisfies (A) and the first row is not α row_1 (linearly dependent).

Using these in (B)

$1 + 1 = b_1 = 2$

$1 - 1 = b_2 = 0$

$$\therefore \begin{pmatrix}1 & 1 & -3\\1 & -1 & 1\end{pmatrix} x = \begin{pmatrix}2\\0\end{pmatrix}$$