HW2

(1) Show that $T: \mathbb{R}^n \to \mathbb{R}^m$ can be done by a matrix multiplication.

(2) Find a basis for the vector space of $\mathbb{R}^{3\times 2}$ (matrices of dimension 3×2 of reals). What is the dimension of $\mathbb{R}^{3\times 2}$?

(3) (a) Show that $A \in \mathbb{R}^{n\times n}$ can be written as

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T) = C + D$$

(b) Show that C is symmetric and D is antisymmetric.

(c) Show that matrices $C \in$ subspace of $\mathbb{R}^{n\times n}$

(d) Show that matrices $D \in$ subspace of $\mathbb{R}^{n\times n}$

(4) Let $z = x + iy \in \mathbb{C}$ complex number

(a) Show that $\mathbb{C}$ is a vector space

(b) Let $T: \mathbb{C} \to \mathbb{C} \mid T(z) = \alpha z$, $\alpha \in \mathbb{C}$

What is the linear transformation T?

(c) The alternative is to treat $\mathbb{C}$ numbers as vectors $\begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2$. What is the linear transformation on $\begin{pmatrix} x \\ y \end{pmatrix}$ that accomplishes multiplication by $\alpha = a + ib$?

(5) For each linear transformation, find the matrix transformation:

(a) $T: \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+2y \\ 2x-5y \\ 7y \end{pmatrix}$

(b) $T: P_n \to P_{n-1}$, where $T(f(x)) = \frac{df}{dx}$, $x \in \mathbb{R}$

(find a matrix with respect to the basis $(1, x, x^2, \ldots, x^n)$)

Take $n=3$ and confirm $T: P_n \to P_{n-1}$

(6) Find the matrix of the reflection through the line $y = -\frac{2x}{3}$ in $\mathbb{R}^2$. Where does $(1,1)$ map to?

(7) Prove that for $A, B \in \mathbb{R}^{n \times n}$

$$\text{Trace}(AB) = \text{Trace}(BA)$$

(8) Let X & Y be subspaces of V. Prove that $X \cap Y$ is a subspace of V.

(9) Let X be a subspace of V, let $\underset{\sim}{v} \in V$, $\underset{\sim}{v} \notin X$. Prove that if $\underset{\sim}{x} \in X$ that $\underset{\sim}{x} + \underset{\sim}{v} \notin X$.

(10) Show that $T: p(x) = \int_{-\infty}^{x} f(s)\,ds \mid f(s), p(x)$ are continuous functions, is a linear operator.

(11) Determine whether $T\begin{pmatrix} x_0 \\ x_1 \end{pmatrix} = \begin{pmatrix} 1 \\ x_0 \end{pmatrix}$ is a linear transformation.