HW1

1. Verify that each of the following is a vector space

(a) 2x2 matrices of the form
$$\left\{\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \middle| a,b\in\mathbb{R}\right\}$$

(b) The collection of $\{a_0 + a_1 x + a_3 x^3\}$, where $a_i \in \mathbb{R}$

2. Determine if each set is linearly independent

(a) $\left\{\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}\right\}$

(b) $\{(1,3,1), (-1,4,3), (-1,11,7)\}$

(c) $\left\{\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ -1 & 4 \end{pmatrix}\right\}$

3. Prove or disprove: there exists a basis (p_0, p_1, p_2, p_3) of $P_3(\mathbb{R})$ such that none of the polynomials p_0, p_1, p_2, p_3 are of degree 2.

4. Construct a basis for $\mathbb{R}^\infty$

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(5) Give 2 different basis for $\mathbb{R}^3$. Verify that each is a basis.

(6) Represent the vector $\underset{\sim}{v} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ with respect to each of the 2 bases

$$B_1 = \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\} \qquad B_2 = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right\}$$

(7) Give a basis for the span of each set

(a) $\left\{ \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 12 \\ 6 \end{pmatrix} \right\}$

(b) $\{ x + x^2, 2 - 2x, 7, 4 + 3x + 2x^2 \}$