NAME:

Instructions: You must include all the steps in your derivations/answers. Reduce answers as much as possible, but use *exact arithmetic*. Write neatly.

1. (10%)

• Given

$$A = \left(\begin{array}{rr} 1 & 1\\ 1 & 0 \end{array}\right)$$

compute A^2 , A^3 , and A^4 . These are called the Fibonacci matrices. What is A^{100} ?

• Let

$$B = \left(\begin{array}{cc} 7 & 12\\ -4 & -7 \end{array}\right)$$

find B^{-101}

2. (10%) Find y_k and z_k , for some k, and find these as $k \to \infty$. The system is

$$\begin{array}{rcl} y_{k+1} &=& 0.8 y_k + 0.3 z_k, \\ z_{k+1} &=& 0.2 y_k + 0.7 z_k, \quad k=0,1,2,\ldots \end{array}$$

 $y_0 = 0$, and $z_0 = 5$.

3. (10%)

• Find all of the eigenvalues and eigenvectors of

- Find a Diagonalization $A = U\Lambda U^{-1}$. Write two different U.
- 4. (10 %) Find all vectors that are perpendicular to (1, 3, 1) and (2, 2, 2)
- 5. (10 %) Find the dimension and basis for the 4 fundamental subspaces of the matrix

$$A = \left(\begin{array}{rrrrr} 1 & 1 & 1 & 1 \\ 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right)$$