HW7 Solutions

(1) $\det AB = \det A \det B = -12$; $\det 5A = 5 \det A = 20$, $\det B^T = -3$
$\det A^{-1} = 1/4$, $\det A^3 = 64$

(2) Form $(A+3I)\underline{x} = 0$ or $\begin{pmatrix} 2 & 4 \\ 6 & 12 \end{pmatrix} \underline{x} = 0$, colums are LD
$\therefore$ non-trivial solution $\therefore$ $\lambda = -3$ is an e'value.

(3) Is Ax a multiple of x? $\begin{bmatrix} 5 & 2 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix} = 3\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

so $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ is e'vector.

(4) $\lambda_1 = 1$ $(A-I)\underline{x} = \emptyset$ is $3x_1 + x_3 = 0$
$-2x_1 = 0$

so $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ works.

$\lambda_2 = 2$ $\begin{bmatrix} -1/2 \\ 1 \\ 1 \end{bmatrix}$ for $\lambda = 3$ $\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

(5) $\lambda = -5$ $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ fr, $\lambda = 2$ $\begin{bmatrix} -0.296 \\ -0.888 \end{bmatrix}$

(6) $A^{-1}Ax = A^{-1}(\lambda \cancel{x})$ so $\underline{x} = \lambda(A^{-1}\underline{x})$ since $\underline{x} \neq 0$
$\lambda \neq 0$. Then $\lambda^{-1}\underline{x} = A^{-1}\underline{x}$ shows that

λ^{-1} is e'value of A^{-1}

(7) $\lambda_{1,2} = 2 \pm i \quad V_{1,2} = \begin{bmatrix} -1 \pm i \\ 1 \end{bmatrix}$

(8) $\lambda = 5, 0, 3$

(9) $\det(A^T - \lambda I) = \det(A^T - \lambda I^T) = \det(A - \lambda I)^T$
$= \det(A - \lambda I)$

(10) (c) $\det(A - \lambda I) = 0$ leads to $\lambda_{1,2} = \begin{cases} 1 \\ 0.92 \end{cases}$

$\underset{\sim}{v}_1 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ associated with $\lambda = 1$

$\underset{\sim}{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ associated with $\lambda = 0.92$

$$\underset{\sim}{x}_0 = \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix} = c_1 \underset{\sim}{v}_1 + c_2 \underset{\sim}{v}_2 \Rightarrow \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} \underset{\sim}{v}_1 & \underset{\sim}{v}_2 \end{bmatrix}^{-1} \underset{\sim}{x}_0 = \begin{bmatrix} 0.125 \\ 0.225 \end{bmatrix}$$

$$\underset{\sim}{x}_1 = A\underset{\sim}{x}_0 = c_1 A\underset{\sim}{v}_1 + c_2 A\underset{\sim}{v}_2 = c_1 \lambda_1 \underset{\sim}{v}_1 + c_2 \lambda_2 \underset{\sim}{v}_2 = c_1 \underset{\sim}{v}_1 + c_2 0.92 \underset{\sim}{v}_2$$

$$\underset{\sim}{x}_2 = A\underset{\sim}{x}_1 = c_1 \underset{\sim}{v}_1 + c_2 (0.92)^2 \underset{\sim}{v}_2$$

$$\vdots$$

$$\underset{\sim}{x}_k = c_1 \underset{\sim}{v}_1 + c_2 (0.92)^k \underset{\sim}{v}_2 \quad k = 0, 1, 2 \ldots$$

$$= 0.125 \begin{bmatrix} 3 \\ 5 \end{bmatrix} + 0.225 (0.92)^k \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad k = 0, 1, \ldots$$

$$\lim_{k \to \infty} \underset{\sim}{x}_k = 0.125 \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 0.375 \\ 0.625 \end{bmatrix} = 0.125 \underset{\sim}{v}_1$$