HW7

1. Let A & B be $\mathbb{R}^{3\times3}$, with $\det A = 4$ and $\det B = -3$. Find
(a) $\det(AB)$, (b) $\det 5A$, (c) $\det B^T$, (d) $\det A^{-1}$, (e) $\det(A^3)$

2. Is $\lambda = -3$ an eigenvalue of $\begin{bmatrix} -1 & 4 \\ 6 & 9 \end{bmatrix}$ Why? why not?

3. Is $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ an eigenvector of $\begin{bmatrix} 5 & 2 \\ 3 & 6 \end{bmatrix}$? if so, find eigenvalue.

4. Find eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

5. Find eigenvalues and eigenvectors of

$$A = \begin{bmatrix} -4 & 2 \\ 3 & 1 \end{bmatrix}$$

6. Let λ be an eigenvalue of an invertible matrix. Show that λ^{-1} is an eigenvalue of A^{-1}
hint: suppose a non-zero vector $\underset{\sim}{x}$ satisfies $A\underset{\sim}{x} = \lambda \underset{\sim}{x}$

7. Find eigenvalues and eigenvectors of

$$\begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$$

8. What are the eigenvalues of

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 2 & 3 \end{bmatrix}$$

(upper/lower) triangular matrices have diagonal elements equal to the eigenvalues.

9. Use the properties of determinants to show that A and A^T have the same characteristic polynomial

10. let $A = \begin{bmatrix} 0.95 & 0.03 \\ 0.05 & 0.97 \end{bmatrix}$

$$\underset{\sim}{x}_0 = \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}$$

(a) Find e'vectors/values of A: $\underset{\sim}{v}_1, \underset{\sim}{v}_2$ and λ_1, λ_2.

Write $\underset{\sim}{x}_0 = c_1 \underset{\sim}{v}_1 + c_2 \underset{\sim}{v}_2$

solve for c_1 & c_2

Investigate the behavior of the system

$$\underset{\sim}{x}_{k+1} = A \underset{\sim}{x}_k \qquad k = 0, 1, 2 \ldots$$

To do so,

(b) for $k=1$ $\quad \underset{\sim}{x}_1 = A \underset{\sim}{x}_0$

write $\underset{\sim}{x}_1$ in terms of $\underset{\sim}{v}_1$ and $\underset{\sim}{v}_2$

hint $A \underset{\sim}{x}_0 = c_1 \lambda_1 \underset{\sim}{v}_1 + c_2 \lambda_2 \underset{\sim}{v}_2$

(c) find $\underset{\sim}{x}_2 = A \underset{\sim}{x}_1 = A (A \underset{\sim}{x}_0)$

(d) by induction, show that

$$\underset{\sim}{x}_n = A^n \underset{\sim}{x}_0$$

(e) now take $\lim_{n \to \infty} \underset{\sim}{x}_n = \underset{\sim}{x}_\infty$

compare $\underset{\sim}{x}_\infty$ and $\underset{\sim}{v}_1$ and $\underset{\sim}{v}_2$.

What do you see?