HW6 Solution

(1) Solve $Ly=b$ to get $y=\begin{bmatrix}2\\0\\6\end{bmatrix}$

then solve $Ux=y$ to get $x=\begin{bmatrix}-11\\-6\\-3\end{bmatrix}$

(2) Row reduce $[L \mid I] \sim \left[\begin{array}{c|ccc} & 1 & 0 & 0 \\ I & 2 & 1 & 0 \\ & -2 & -1 & 0\end{array}\right]$

Row reduce $[U \mid I] \sim \left[\begin{array}{c|ccc} & 1/2 & -3/4 & -2 \\ I & 0 & -1/4 & -1 \\ & 0 & 0 & -1/2\end{array}\right]$

$A^{-1}=U^{-1}L^{-1}=\begin{bmatrix}3 & 5/4 & -2\\ 3/2 & 3/4 & -1\\ 1 & 1/2 & -1/2\end{bmatrix}$

(3) Yes. $A=\begin{bmatrix}0 & 5 & 6\\ 0 & 0 & 3\\ -2 & 4 & 2\end{bmatrix} \sim \begin{bmatrix}-2 & 4 & 2\\ 0 & 5 & 6\\ 0 & 0 & 3\end{bmatrix}$ 3 pivots $\therefore$ LI and A^{-1} exists.

and vectors are basis of $\mathbb{R}^3$.

(4)

$[A \mid p]=\begin{bmatrix}-2 & -2 & 0 & -6\\ 0 & 3 & -5 & 1\\ 6 & 3 & 5 & 17\end{bmatrix} \sim \begin{bmatrix}-2 & -2 & 0 & -6\\ 0 & 3 & -5 & 1\\ 0 & 0 & 0 & 0\end{bmatrix}$

Consistent $\therefore p \in \mathrm{Col}(A)$

If $\underset{\sim}{u} \in \mathrm{Null}(A)$ then $A\underset{\sim}{u}=0$, which checks out.

(5) $\begin{bmatrix} 1 & -3 & 2 & -4 \\ -3 & 9 & 1 & 5 \\ 2 & -6 & 4 & -3 \\ -4 & 12 & 2 & 7 \end{bmatrix} \sim \begin{bmatrix} \underline{1} & -3 & 2 & -4 \\ 0 & 0 & \underline{5} & -7 \\ 0 & 0 & 0 & \underline{5} \\ 0 & 0 & 0 & 0 \end{bmatrix}$

So $col(A) = col_1, col_3, col_4$ $\quad$ $\dim(col A) = 3$.

(6) $Col(A) = \{col_1, col_3, col_4\} = \left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 5 \\ 9 \\ 14 \end{bmatrix} \right\}$

(7) $A \in \mathbb{R}^{6 \times 8}$ $\quad$ $rank(A) = 8 - \dim(Null(A)) = 8 - 3 = 5$

(8) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix}$, for example. Cols 2, 3, 4 are multiples of Col 1.

(9) $H = span\left\{ \begin{bmatrix} 2 \\ -5 \\ -3 \end{bmatrix} \right\} \Rightarrow H$ is subspace of $\mathbb{R}^3$, $t = -1$ for example.

(10) $[A | 0] \sim \left[\begin{array}{cccc|c} 1 & 0 & -2 & 4 & 0 \\ 0 & 1 & 3 & -2 & 0 \end{array}\right] \Rightarrow \begin{array}{l} x_1 = 2x_3 - 4x_4 \\ x_2 = -3x_3 + 2x_4 \end{array}$ $\quad x_3, x_4$ free.

So $\underset{\sim}{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} 2 \\ -3 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -4 \\ 2 \\ 0 \\ 1 \end{bmatrix}$

vectors span Null(A)

(11) Not a vector space since $\emptyset$ is not in W.

(12) $A = [1 \ -3 \ 2]$ To find null(A) find $A\underset{\sim}{x} = \emptyset$

$$\underset{\sim}{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = y\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + z\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$ (y, z free)

Null(A)

(13) Each set $\{\underset{\sim}{v}_1, \underset{\sim}{v}_2\}$, $\{\underline{v}_1, \underline{v}_3\}$, $\{\underline{v}_2, \underline{v}_3\}$

span H, because pairwise they are LI. But v_1, v_2, v_3

are L Dependent $\therefore$ $H \neq \{\underline{v}_1, \underline{v}_2, \underline{v}_3\}$

(14) $\underset{\sim}{v}_1$ & $\underline{v}_2$ are LI So $H = \text{span}\{v_1, v_2\}$

has $\dim(H) = 2$

(15) Since A has 4 pivots $\text{rank}(A) = 4$, $\dim(\text{Null}(A)) = 0$
Yes $\text{col}(A) = \mathbb{R}^4$.