HW6

1. Solve $A\underset{\sim}{x} = \underset{\sim}{b}$ using LU factorization

$$A = \begin{bmatrix} 2 & -6 & 4 \\ -4 & 8 & 0 \\ 0 & -4 & 6 \end{bmatrix}, \quad \underset{\sim}{b} = \begin{bmatrix} 2 \\ -4 \\ 6 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -6 & 4 \\ 0 & -4 & 8 \\ 0 & 0 & -2 \end{bmatrix}$$

2. Find A^{-1} by computing $U^{-1}L^{-1}$ for A from 1.

3. Determine whether the following are basis for $\mathbb{R}^1$, $\mathbb{R}^2$, or $\mathbb{R}^3$:

$$\begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

4. Let $A = \begin{bmatrix} -2 & -2 & 0 \\ 0 & 3 & -5 \\ 6 & 3 & 5 \end{bmatrix}$, $\underset{\sim}{p} = \begin{bmatrix} -6 \\ 1 \\ 17 \end{bmatrix}$, $\underset{\sim}{u} = \begin{bmatrix} -5 \\ 5 \\ 3 \end{bmatrix}$

(a) Determine whether $\underset{\sim}{p}$ is in $\mathrm{Col}(A)$

(b) Determine whether $\underset{\sim}{u}$ is in $\mathrm{Null}(A)$

⑤ Find a basis for the subspace spanned by

$$\begin{bmatrix} 1 \\ -3 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} -3 \\ 9 \\ -6 \\ 12 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} -4 \\ 5 \\ -3 \\ 7 \end{bmatrix}$$

What's the dimension of the subspace?

⑥ If $A = \begin{bmatrix} 1 & 3 & 2 & -6 \\ 3 & 9 & 1 & 5 \\ 2 & 6 & -1 & 9 \\ 5 & 15 & 0 & 14 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 3 & 2 \\ 0 & 0 & 5 & -7 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

find basis for $Col(A)$ and $Null(A)$ and their dimensions.

⑦ What is the rank of a 6×8 matrix whose null space is of dimension 3?

⑧ Construct a 3×4 matrix of rank 1.

⑨ Let H be the set of all vectors of the form $\begin{bmatrix} -2t \\ 5t \\ 3t \end{bmatrix}$. Find a vector $\underset{\sim}{v}$ in $\mathbb{R}^3$ such that

$$H = \text{span}\{\underset{\sim}{v}\}.$$

(10) Find an explicit description of Null(A) by listing vectors that span the nullspace of A:

$$A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ 0 & 1 & 3 & -2 \end{bmatrix}$$

(11) Is $W = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \text{ such that } a+b+c=2 \right\}$ a vector space? Why/why not?

(12) Find a basis for the set of vectors in $\mathbb{R}^3$ in the plane $x - 3y + 2z = 0$. Hint: think of the equation as a system of homogeneous equations.

(13) Let $\underline{v}_1 = \begin{bmatrix} 3 \\ 4 \\ -2 \\ -5 \end{bmatrix}$ $\underline{v}_2 = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 4 \end{bmatrix}$ and $\underline{v}_3 = \begin{bmatrix} 2 \\ 5 \\ -6 \\ -14 \end{bmatrix}$.

It ~~can~~ be verified that $4\underline{v}_1 + 5\underline{v}_2 - 3\underline{v}_3 = 0$

Let $H = \text{span}\{\underline{v}_1, \underline{v}_2, \underline{v}_3\}$. Find a basis for H.

(14) Find a basis for the subspace and its dimension:

$$\left\{ \begin{bmatrix} s-2t \\ s+t \\ 3t \end{bmatrix} \text{ such that } s,t \in \mathbb{R} \right\}$$

(15) Suppose 4x7 matrix has 4 pivots. What's the dimension of its null space? Is Col the matrix $\mathbb{R}^4$ why?/why not?