HW5 Solutions

① let $B = [\underset{-}{b_1}\ \underset{-}{b_2}] \Rightarrow AB = \phi$ is $A\underset{-}{b_1} = 0$ and $A\underset{-}{b_2} = 0$

or $3b_1^1 - 6b_1^2 = 0$ so $\underset{-}{b_1} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ or any multiple of it

$-2b_1^1 + 4b_1^2 = 0$

$A\underset{-}{b_2} = 0$ is a similar system. So $b_2 = x b_1$, where x is a scalar.

$\therefore B = \begin{bmatrix} 2 & 8 \\ 1 & 4 \end{bmatrix}$, say.

② $\begin{bmatrix} -3 & -11 \\ 1 & 17 \end{bmatrix} = AB = \begin{bmatrix} A\underset{-}{b_1} & A\underset{-}{b_2} \end{bmatrix}$, using $A = \begin{bmatrix} 1 & -3 \\ -3 & 5 \end{bmatrix}$

we see $Ab_1 = \begin{bmatrix} -3 \\ 1 \end{bmatrix} \Rightarrow \left[\begin{array}{cc|c} 1 & -3 & -3 \\ -3 & 5 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \end{array}\right]$

so $b_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$. Same process for $b_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

③ $u^Tv \in \mathbb{R}^1$, $v^Tu \in \mathbb{R}^1$, $uv^T \in \mathbb{R}^{n\times n}$, $vu^T \in \mathbb{R}^{n\times n}$

④ $AX = B$ with $A \in \mathbb{R}^{n\times n}$ invertible, $B \in \mathbb{R}^{n\times p}$ is equivalent to p problems

$Ax_i = b_i$, where $x_i \in \mathbb{R}^n$, $b_i \in \mathbb{R}^n$ and $i = 1, 2, .., p$.

Since A is invertible $x_i = A^{-1}b_i$, unique for $i = 1, 2 \ldots p$.

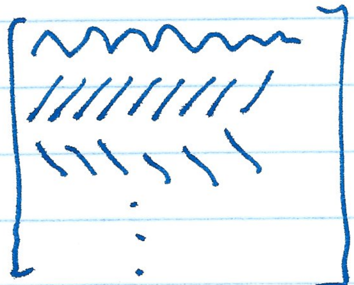
(5) $AB = AC \Rightarrow A^{-1}AB = A^{-1}AC \Rightarrow B = C$

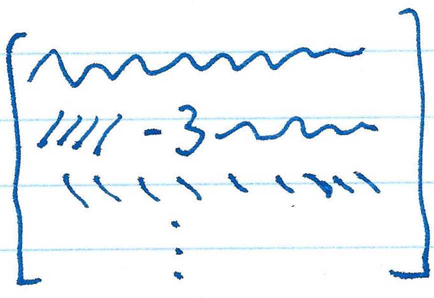
(6) $C(A-AC)^{-1} = B$ and B is a product of 2 invertible matrices.

(b) $A(A-AC)^{-1} = C^{-1}B \Rightarrow I = C^{-1}B(A-AC)$

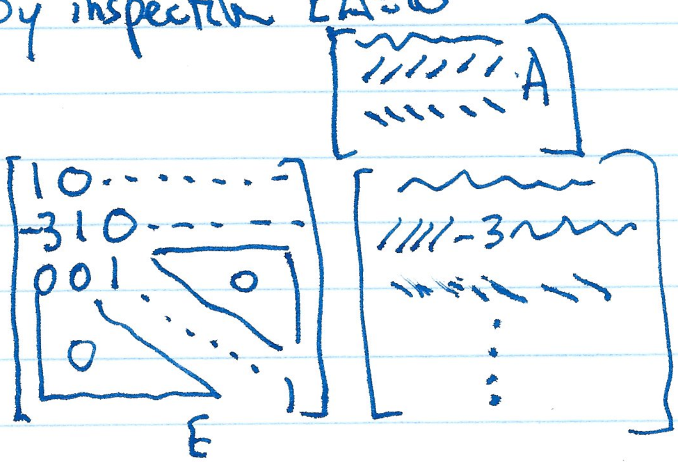
$\Rightarrow (C^{-1}B)^{-1} = A-AC \Rightarrow B^{-1}C = A-AC$

$\Rightarrow (B^{-1}+A)C = A \Rightarrow C = (B^{-1}+A)^{-1}A = BA+I$

(7) $EA = B$ where $A =$ 

and $B =$ 

by inspection $EA = B$



8. $$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 1 & 7/2 & 3/2 & 1/2 \end{array}\right]$$

9. $$A = \begin{bmatrix} 3 & 4 & 7 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix}$$ find A^{-1} setup $AA^{-1} = I$

$$\begin{bmatrix} & \text{———} & \\ 0 & 1 & -2 \\ 0 & 0 & 1/2 \end{bmatrix}$$

$$A^{-1} \Rightarrow \begin{bmatrix} & \text{———} & \\ & \text{———} & \\ 0 & 0 & 1/2 \end{bmatrix}$$

(a)
$$\begin{bmatrix} 3 & 4 & 7 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} & \text{———} & \\ & \text{———} & \\ 0 & 0 & 1 \end{bmatrix}$$
$\uparrow A$

(b)
$$\begin{bmatrix} 3 & 4 & 7 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} & \text{———} & \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1/3 & -4/3 & 3/2 \\ 0 & 1 & -2 \\ 0 & 0 & 1/2 \end{bmatrix} \Longleftarrow A^{-1}$$

(c)
$$\begin{bmatrix} 3 & 4 & 7 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(10) No. The row echelon procedure will produce 1 pivot less than the dimension of the matrix.

(11) AB is invertible A & B $n \times n$ matrices

let $C = (AB)^{-1}$ $\Rightarrow$ $CAB = I$ or $CA = B^{-1}$

$\therefore$ B^{-1} exists.

(12) If $T(\underset{\sim}{u}) = T(\underset{\sim}{v})$ then T is not producing, for $\underset{\sim}{u} \neq \underset{\sim}{v}$, unique images in $\mathbb{R}^n$

This implies that T is not one-to-one and has linearly dependent columns. Thus T cannot map $\mathbb{R}^n \to \mathbb{R}^n$.