HW 5

① Let $A = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$ Construct a 2×2 matrix B such that AB is the zero matrix. Use 2 different nonzero columns for B!

② If $A = \begin{bmatrix} 1 & -3 \\ -3 & 5 \end{bmatrix}$ and $AB = \begin{bmatrix} -3 & -11 \\ 1 & 17 \end{bmatrix}$ determine B.

③ If $\underset{\sim}{u}$ and $\underset{\sim}{v} \in \mathbb{R}^n$, what is the dimension of $\underline{u}^T\underline{v}$, $\underline{v}^T\underline{u}$, uv^T and vu^T?

④ Let $A \in \mathbb{R}^{n\times n}$ invertible matrix. Let $B \in \mathbb{R}^{n\times p}$ matrix. Show that $AX = B$ has a unique solution $X = A^{-1}B$.

⑤ Suppose $AB = AC$ where B & $C \in \mathbb{R}^{n\times p}$ and A is invertible. Show that $B = C$. Is this true, in general, if A is not invertible?

⑥ If A, B & C are $n\times n$ matrices with A, C and $A - AC$ invertible and suppose

$$(A - AC)^{-1} = C^{-1}B \quad (*)$$

a) Explain why B is invertible

b) Solve $(*)$ for C.

(7) Suppose row 2 of A is replaced by $\text{row}_2(A) - 3\,\text{row}_1(A)$
Let the resultant matrix be B. Find
E, the matrix that, multiplied by A, leads to B

$$EA = B$$

Ok to assume that $A \in \mathbb{R}^{n \times n}$

(8) Invert $\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$

(9) Invert the ~~diagonal~~ "upper triangular" matrix

$$\begin{bmatrix} 3 & 4 & 7 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix}$$

(upper/lower) triangular matrices are easier to invert. Recognizing this structure saves a lot of work!

Form $AA^{-1} = I$ and work from bottom to top to find A^{-1}

(10) Can a matrix with 2 identical rows be invertible? why/why not?

(11) Let A and B be $n \times n$ matrices. Show that if (AB) is invertible, so is B.

(12) Suppose $T: \mathbb{R}^n \to \mathbb{R}^n$ has the property $T(\underline{u}) = T(\underline{v})$ for some pair of distinct $\mathbb{R}^n$ vectors $\underline{u}$ and $\underline{v}$. Can T map $\mathbb{R}^n$ to $\mathbb{R}^n$? Why?/why not?