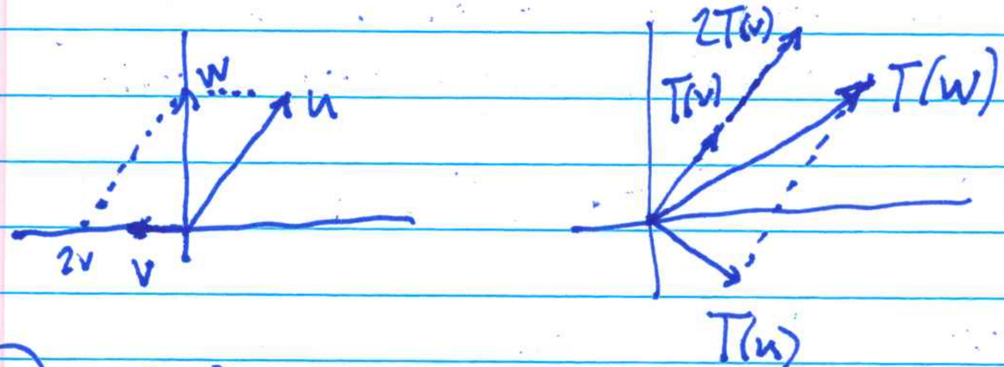
HW 4 Solution

① $a = 5$. The domain of T is $\mathbb{R}^5$, $x \in \mathbb{R}^5$. $b = 6$

The codomain of T is $\mathbb{R}^6$, $Ax \in \mathbb{R}^6$.

② $\underset{\sim}{w} = \underset{\sim}{u} + 2\underset{\sim}{v}$. $T(\underset{\sim}{w}) = T(\underset{\sim}{u}) + 2T(\underset{\sim}{v})$



③ $T(\underset{\sim}{x}) = x_1 \underset{\sim}{v}_1 + x_2 \underset{\sim}{v}_2$

$$= \begin{bmatrix} \underset{\sim}{v}_1 & \underset{\sim}{v}_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 & 7 \\ 5 & -2 \end{bmatrix} \underset{\sim}{x} = A\underset{\sim}{x}$$

④ (a) $b = 0 \Rightarrow f = mx$, linear superposition holds:

$$f(cx + dy) = c f(x) + d f(y)$$

(b) if $b \neq 0 \Rightarrow f(0) = m(0) + b \neq 0$.

every linear transformation maps the $\emptyset$ vector of its domain to the $\emptyset$ of its codomain.

(c) f is a "straight line" a linear function

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(5) since $c_1 \underline{v}_1 + c_2 \underline{v}_2 + c_3 \underline{v}_3 = \phi$.

can be achieved with c_i ~~not~~ all zero,

Then $T(c_1 \underline{v}_1 + c_2 \underline{v}_2 + c_3 \underline{v}_3) = T(\phi)$

$$c_1 T(\underline{v}_1) + c_2 T(\underline{v}_2) + c\, T(\underline{v}_3) = \phi$$

since T is linear. Since c_i not all zero, $\{T(v_1), T(v_2), T(v_3)\}$ Linearly dependent.

(6) Using $V = IR$ and $\sum V_i = 0$ in each loop:

loop 1: $40 - 1I_1 - 7(I_1 - I_2) - 4(I_1 - I_4) = 0$

loop 2: $30 - 2I_2 - 6(I_2 - I_3) - 7(I_2 - I_1) = 0$

loop 3 $20 - 3I_3 - 5(I_3 - I_4) - 6(I_3 - I_2) = 0$

loop 4 $+10 - 4I_4 - 4(I_4 - I_1) - 5(I_4 - I_3) = 0$

let $\underline{x} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$ $\underline{b} = \begin{bmatrix} 40 \\ 30 \\ 20 \\ +10 \end{bmatrix}$ $A = \begin{bmatrix} 12 & -7 & 0 & -4 \\ -7 & 15 & -6 & 0 \\ 0 & -6 & 14 & -5 \\ 4 & 0 & 5 & -13 \end{bmatrix}$

Solving $x = A^{-1} b = [11.43, 10.55, 8.04, 5.84]^T$.

(7)

From:

City	Suburbs	To
1-0.06	0.04	City
0.06	1-0.04	Suburbs

$$M = \begin{bmatrix} 0.94 & 0.04 \\ 0.06 & 0.96 \end{bmatrix} \qquad \underset{\sim}{x_0} = \begin{bmatrix} 10^7 \\ 8\cdot 10^5 \end{bmatrix}$$

$$\underset{\sim}{x_2} = M\underset{\sim}{x_1} = M(M\underset{\sim}{x_0}) = \begin{bmatrix} 8920800 \\ 1879200 \end{bmatrix}$$

$$\underset{\sim}{x_{20}} = M^{20}\underset{\sim}{x_0} = \begin{bmatrix} 5.011 \\ 5.800 \end{bmatrix}\cdot 10^6$$

more people in suburbs than city.

(8)

$$H_3O: \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad CaCO_3: \begin{bmatrix} 0 \\ 3 \\ 1 \\ 1 \end{bmatrix} \quad H_2O: \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad Ca: \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad CO_2: \begin{array}{cc} \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} & \begin{matrix} H \\ O \\ Ca \\ C \end{matrix} \end{array}$$

$$\therefore \quad x_1 \cdot H_3O + x_2 \cdot CaCO_3 \rightarrow x_3 \cdot H_2O + x_4\, Ca + x_5\, CO_2$$

or $$x_1\begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_2\begin{bmatrix} 0 \\ 3 \\ 1 \\ 1 \end{bmatrix} = x_3\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_5\begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}.$$

$$\begin{bmatrix} 3 & 0 & -2 & 0 & 0 & 0 \\ 1 & 3 & -1 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

The general solution is

$x_1 = 2x_5$

$x_2 = x_5$

$x_3 = 3x_5$

$x_4 = x_5$

x_5 free.

Take $x_5 = 1 \Rightarrow x_1 = 2, x_2 = x_4 = 1, x_3 = 3$

$$2\,H_3O + CaCO_3 \rightarrow 3H_2O + Ca + CO_2$$

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Output			Purchased by
F	M	S	
0.1	0.1	0.2	F
0.8	0.1	0.4	M
0.1	0.8	0.4	S
Total: 1	1	1	

(b) productivity vector $\underline{P} = (p_F, p_M, p_S)$

From Table:

row 1 $\quad p_F = 0.1\,p_F + 0.1\,p_M + 0.2\,p_S$

row 2 $\quad p_M = 0.8\,p_F + 0.1\,p_M + 0.4\,p_S$

row 3 $\quad p_S = 0.1\,p_F + 0.8\,p_M + 0.4\,p_S$

or $\begin{bmatrix} 0.9 & -0.1 & -0.2 \\ -0.8 & 0.9 & -0.4 \\ -0.1 & -0.8 & 0.6 \end{bmatrix} \underline{P} = \phi$ the row reduced augmented system

(c)

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -0.301 & 0 \\ 0 & 1 & -0.712 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore$ p_S arbitrary. $\quad p_F = 0.301\,p_S \quad p_M = 0.712\,p_S$

if $p_S = 100\%$, $p_F = 30\%$ $\quad p_M = 71\%$

Are the ratios, with respect to p_S.