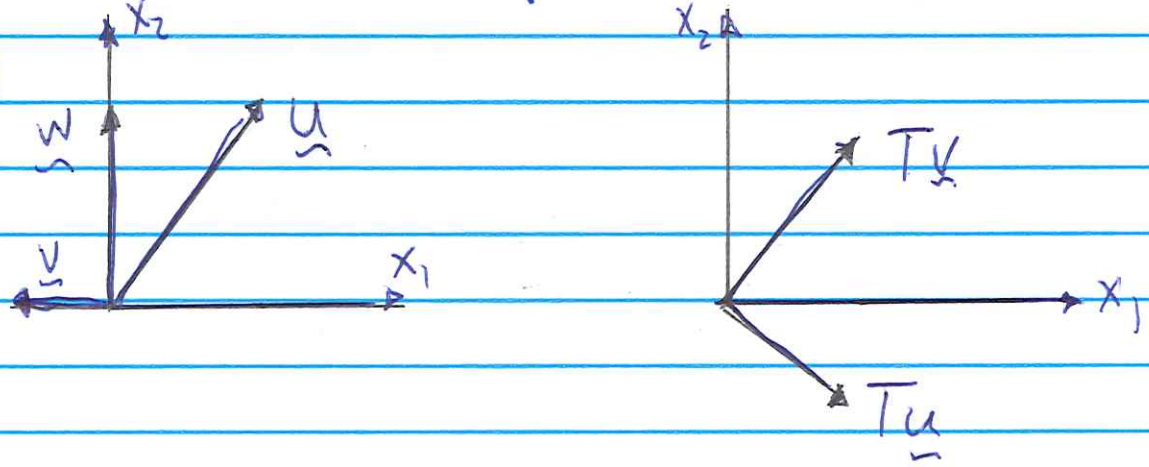
HW 8

(1) Let A be a 6×5 matrix. What must a and b be in order to define the linear map $T: \mathbb{R}^a \to \mathbb{R}^b$ by $T(\underset{\sim}{x}) = A\underset{\sim}{x}$?

(2)



The figure shows the vector $\underset{\sim}{u}, \underset{\sim}{v}, \underset{\sim}{w}$, along with images $T\underset{\sim}{u}$ and $T\underset{\sim}{v}$. T is a linear transformation from $\mathbb{R}^2 \to \mathbb{R}^2$. Copy figure and draw $T\underset{\sim}{w}$ as carefully as possible.

(3) Let $\underset{\sim}{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $\underset{\sim}{v}_1 = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$ and $\underset{\sim}{v}_2 = \begin{bmatrix} 7 \\ -2 \end{bmatrix}$

and let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that map $\underset{\sim}{x}$ into $x_1\underset{\sim}{v}_1 + x_2\underset{\sim}{v}_2$. Find a matrix A such that $T(\underset{\sim}{x})$ is $A\underset{\sim}{x}$ for each $\underset{\sim}{x}$.

(4) Define $f: \mathbb{R} \to \mathbb{R}$ by $f(x) = mx + b$.

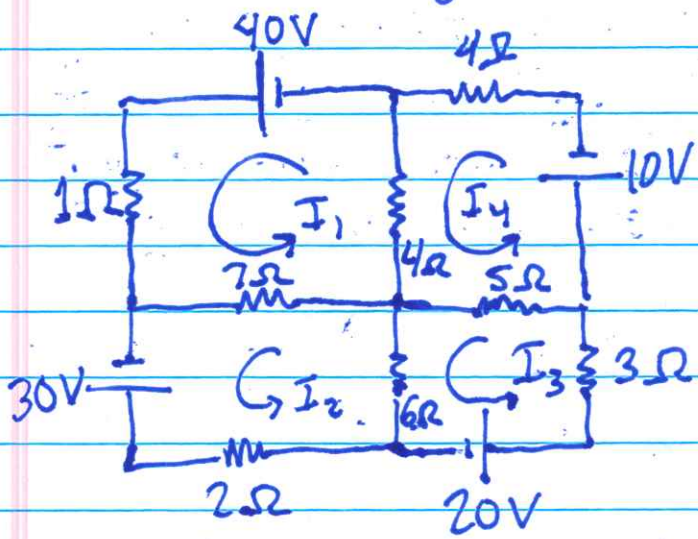
(a) Show that f is a linear transformation when $b = 0$.

(b) Find a property of a linear transformation that is violated when $b \neq 0$.

(c) Why is f called a linear function?

(5) Let $T: \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation and let $\{\underline{v}_1, \underline{v}_2, \underline{v}_3\}$ be a linearly dependent set in $\mathbb{R}^n$. Explain why the set $\{T(\underline{v}_1), T(\underline{v}_2), T(\underline{v}_3)\}$ is linearly dependent.

(6) Determine the loop currents I_1, I_2, I_3, I_4 in the following circuit using Ohm's Law:



(7) In a certain region, about 6% of a city's population moves to the surrounding suburbs each year. About 4% of the suburban population moves into the city. In 2017, there were 10 Million residents in ~~there~~ city and 800 thousand in the suburbs. Set up a difference equation that describes this

situation, where x_0 is the population in 2017. Then estimate the population 2 years and 20 years later.

(8) Balance the chemical equation: limestone $CaCO_3$ neutralizes the acid H_3O in acid rain (carbolic acid) by the following unbalanced equation:

$$H_3O + CaCO_3 \rightarrow H_2O + Ca + CO_2$$

(9) Consider an economy with 3 sectors: fuels and power, manufacturing, and services.

* Fuel & power sells 80% of it output to Manufacturing, 10% to Services, and retains the rest.
* Manufacturing sells 10% of its output to Fuels & Power, 80% to Services, and retains the rest.
* Services sells 20% to fuels & power, 40% to manufacturing, and retains the rest.

(a) Construct the exchange table for this economy.
(b) Develop a system of equations that leads to prices at which each sector's income matches its expenses. Then, write the augment matrix that can be row reduced to find these prices.
(c) Find a set of equilibrium prices when the price for the services output is 100 units.