HW3 Solutions

(2) $\begin{bmatrix} 3 & -2 \\ 1 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, by inspection.

(3) $A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 - 5x_2 + 4x_3 \\ x_2 - 6x_3 \end{bmatrix} \quad \therefore A = \begin{bmatrix} 1 & -5 & 4 \\ 0 & 1 & -6 \end{bmatrix}$

(4) (a) True, (b) True, (c) False

(a) $I = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix} \in \mathbb{R}^{n \times n}$

can be written as $I = [\hat{e}_1, \hat{e}_2, \dots, \hat{e}_n]$

$\hat{e}_i \in \begin{pmatrix} 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{pmatrix}$ i^{th} entry, the "standard basis"

$\underset{\sim}{x} = c_1\hat{e}_1 + c_2\hat{e}_2 \cdots + c_n\hat{e}_n$

So $T(x) = Ax = [T(\hat{e}_1) \; T(\hat{e}_2) \cdots T(\hat{e}_n)] \underset{\sim}{c}$

(b) $T(0) = 0$, $T(\underline{x} + \underline{y}) = T\underline{x} + T\underline{y}$, $T(x) + T(-x) = \emptyset$.

(c) $TSx = Tv$ may not be the same as $STx = Sw$
unless $[TS - ST]y = 0$