HW2

(1) Write the matrix form $Ax=b$ for the following

$$x_1\begin{bmatrix}4\\-1\\7\\-4\end{bmatrix}+x_2\begin{bmatrix}-5\\3\\-5\\1\end{bmatrix}+x_3\begin{bmatrix}7\\-8\\0\\2\end{bmatrix}=\begin{bmatrix}6\\-8\\0\\-7\end{bmatrix}$$

(2) Let $\underset{\sim}{u}=\begin{bmatrix}0\\4\\4\end{bmatrix}$ and $A=\begin{bmatrix}3&-5\\-2&6\\1&1\end{bmatrix}=\begin{bmatrix}\underset{-}{a}_1 & \underset{-}{a}_2\\ & \end{bmatrix}$ $\underset{-}{a}_i$ are columns of A

(a) Make a figure in $\mathbb{R}^3$ of $\underset{\sim}{u}, \underset{-}{a}_1, \underset{-}{a}_2$.

(b) Determine whether $\underset{\sim}{u}$ is in the plane spanned by $\underset{\sim}{a}_1$ and $\underset{\sim}{a}_2$

(3) Let $A=\begin{bmatrix}1&-2&-1\\-2&2&0\\4&-1&3\end{bmatrix}$ and $\underset{\sim}{b}=\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}$

(a) Show for what values of b_1, b_2, b_3 the system $Ax=b$ has a solution.

(4) Let $\underset{-}{v}_1=\begin{bmatrix}0\\0\\-3\end{bmatrix}$, $\underset{-}{v}_2=\begin{bmatrix}0\\-3\\9\end{bmatrix}$, $\underset{\sim}{v}_3=\begin{bmatrix}4\\-2\\-6\end{bmatrix}$

Does $\{\underset{-}{v}_1, \underset{-}{v}_2, \underset{-}{v}_3\}$ span $\mathbb{R}^3$? Why or why not?

(5) Determine if the system has a nontrivial solution:

$$\begin{aligned} x_1 - 2x_2 + 3x_3 &= 0 \\ -2x_1 - 3x_2 - 4x_3 &= 0 \\ 2x_1 - 4x_2 + 9x_3 &= 0 \end{aligned}$$

(6) Describe all solutions of $A\underline{x} = \emptyset$, where A is row-equivalent to

$$\begin{bmatrix} 1 & 3 & -3 & 7 \\ 0 & 1 & -4 & 5 \end{bmatrix}$$

(7) Given $A = \begin{bmatrix} 3 & -2 \\ -6 & 4 \\ 12 & -8 \end{bmatrix}$ find one non-trivial solution to $A\underline{x} = \emptyset$ by inspection

(8) Construct a 2x2 matrix A such that the solution of $A\underline{x} = \emptyset$ is the line through $(4,1)$ and the origin. Then find a vector $\underline{b}$ in $\mathbb{R}^2$ parallel to the solution set of

$A\underline{x} = \underline{b}$ that is

is not a line in $\mathbb{R}^2$ parallel to the solution set of $A\underline{x} = \emptyset$. Why does this not contradict

"The general solution to $A\underline{x} = \underline{b}$ is $\underline{x} = \underline{x}_h + \underline{x}_p$ where $A\underline{x}_p = \underline{b}$ and $A\underline{x}_h = \emptyset$"

(9) Determine whether the vectors

$$\begin{bmatrix} 2 \\ -3 \end{bmatrix} \text{ and } \begin{bmatrix} -4 \\ 6 \end{bmatrix}$$ are linearly independent

(10) Consider $\underset{\sim}{v}_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$ $\underline{v}_2 = \begin{bmatrix} -3 \\ 9 \\ -6 \end{bmatrix}$ $\underline{v}_3 = \begin{bmatrix} 5 \\ -7 \\ h \end{bmatrix}$

(a) For what values of h is v_3 in Span$\{\underset{\sim}{v}_1, \underset{\sim}{v}_2\}$

(b) For what values of h are the 3 vectors linearly dependent?

(11) Same question as in (10) for

$$\underline{v}_1 = \begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix} \quad \underline{v}_2 = \begin{pmatrix} -3 \\ 9 \\ 15 \end{pmatrix} \quad \underline{v}_3 = \begin{pmatrix} 2 \\ -5 \\ h \end{pmatrix}$$

(12) True/False and why: If $\{\underset{\sim}{v}_1, \underline{v}_2, \underset{\sim}{v}_3, \underset{\sim}{v}_4\} \in \mathbb{R}^4$ and are linearly independent, then

$\{v_1, v_2, v_3\}$ is also linearly independent. Hint: think about

$$x_1 \underline{v}_1 + x_2 \underline{v}_2 + x_3 \underline{v}_3 + 0 \cdot \underset{\sim}{v}_4 = \emptyset$$