HW1 Solutions

(1) $\left(\begin{array}{ccc|c} 5 & -1 & 2 & 7 \\ -2 & 6 & 9 & 0 \\ -7 & 5 & -3 & -7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1.6154 \\ 0 & 1 & 0 & 0.7692 \\ 0 & 0 & 1 & -0.1538 \end{array}\right)$

to five significant digits, Hence $\begin{pmatrix} 1.6154 \\ 0.7692 \\ -0.1538 \end{pmatrix}$

is the solution of the system, not $(3,4,-2)^T$.

Note: it is sufficient to substitute $(3,4,-2)$ into any of the equations to see that it cannot be a solution.

(2) $\left(\begin{array}{ccc|c} 1 & -3 & 0 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -3 & 0 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{array}\right)$

if solution is $(x_1, x_2, x_3)^T$, then $x_3 = -2$

$x_2 = -5x_3 = 10$, $x_1 = 3x_2 = 30$. HOWEVER, the last

equation says $0x_1 + 0x_2 + 0x_3 = 1$, which indicates that the system is inconsistent $\Rightarrow$ no solution.

(3) $\left(\begin{array}{cccc|c} 1 & 0 & -2 & 0 & -1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & -3 & 2 & 0 & 0 \\ -4 & 0 & 0 & 7 & -5 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2.616 \\ 0 & 1 & 0 & 0 & 1.2105 \\ 0 & 0 & 1 & 0 & 1.8158 \\ 0 & 0 & 0 & 1 & -0.7895 \end{array}\right)$ consistent system & unique solution

(4) Determine k for consistency:

$$\left(\begin{array}{cc|c} 1 & k & -2 \\ -4 & 2 & 10 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & k & -2 \\ 0 & 2k+1 & 1 \end{array}\right) \longrightarrow \text{so } (2k+1)x_2 = 1$$

if $k = -\frac{1}{2}$ system is inconsistent. for $k \neq \frac{1}{2}$ the system is consistent.

(5) Row reduce

$$\begin{bmatrix} 1 & 4 & 7 & 10 \\ 2 & 5 & 8 & 11 \\ 3 & 6 & 9 & 12 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

(6) ~~Find~~ Find general solution to

(a) $\left[\begin{array}{ccc|c} 1 & -3 & 0 & -5 \\ -3 & 7 & 0 & 9 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \end{array}\right] \Rightarrow x_1 = 4, x_3$ arbitrary
$x_2 = 3$

(b) $\left[\begin{array}{cc|c} 1 & 2 & -7 \\ -1 & -1 & 1 \\ 2 & 1 & 5 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$ no solution

(c) No solution.

(7) $\begin{bmatrix} x & x & x & x & x \\ x & x & x & x & y \\ 0 & 0 & 0 & 0 & z \end{bmatrix}$ is the system consistent? No, because $0x_1 + 0x_2 + 0x_3 + 0x_4 = z \neq 0$ cannot be found by any $(x_1, x_2, x_3, x_4)^T$.

(8) System equivalent to

$$x_1 \begin{bmatrix} 3 \\ -2 \\ 8 \end{bmatrix} + x_2 \begin{bmatrix} 5 \\ 0 \\ 9 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 8 \end{bmatrix}, \quad \begin{bmatrix} 3x_1 \\ -2x_1 \\ 8x_1 \end{bmatrix} + \begin{bmatrix} 5x_2 \\ 0 \\ 9x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 8 \end{bmatrix}$$

or $3x_1 + 5x_2 = 2$; $-2x_1 = -3$; $8x_1 + 9x_2 = 8$

(9) Let $a_1 = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$, $a_2 = \begin{bmatrix} -5 \\ -8 \\ 2 \end{bmatrix}$ find $b = \begin{bmatrix} 3 \\ -5 \\ +h \end{bmatrix}$ laying in plane spanned by a_1 & a_2:

$$\left[\begin{array}{cc|c} 1 & -5 & 3 \\ 3 & -8 & -5 \\ -1 & 2 & +h \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -5 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & -h-3 \end{array}\right]$$

if $-h = 3 \Rightarrow b$ is in span of a_1 & a_2

(10) Let $A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}$ $b = \begin{bmatrix} 10 \\ 3 \\ 7 \end{bmatrix}$, let W = linear combinations of colums of A

(a) is b in span$[W]$?

$$\left[\begin{array}{ccc|c} 2 & 0 & 6 & 10 \\ -1 & 8 & 5 & 3 \\ 1 & -2 & 1 & 7 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 3 & 5 \\ 0 & 8 & 8 & 8 \\ 0 & 0 & 0 & 4 \end{array}\right] \Rightarrow$$ b is not a linear combination of W elements.

(b) The second column is in W, because $a_2 = 0 \cdot a_1 + 1 \cdot a_2 + 0 \cdot a_3$.

(11) $Ax = b \Rightarrow \left(\begin{array}{ccc|c} x & x & x & x \\ x & x & x & x \\ x & x & x & x \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & x' & x' & x' \\ 0 & 1 & x' & x' \\ 0 & 0 & 0 & x' \end{array}\right)$