LINEAR INDEPENDENCE:

Let $x_i \in \mathbb{R}^n$ vectors.

if $x_1 \underset{\sim}{v}_1 + x_2 \underset{\sim}{v}_2 + \cdots x_p \underset{\sim}{v}_p = \emptyset$

with all $x_i = 0 \Rightarrow \{v_i\}_{i=1}^{p}$ are LI

Thm $S = \{v_1, \ldots, v_p\}$ are Linearly dependent (LD) iff at least 1 vector is a linear combination of others.

Thm: $S = \{\underline{v}_1, \ldots, \underline{v}_p\}$ & $\underset{\sim}{v}_i \in \mathbb{R}^n$. If $p > n$ then S must be LD

Thm: If $S = \{\underset{\sim}{v}_i\}_{i=1}^{p}$, $\underset{\sim}{v}_i \in \mathbb{R}^n$ if 1 or more $\underset{\sim}{v}_j = \emptyset \Rightarrow$ S is LD set.

ex) For what values of h are $\underset{\sim}{v}_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$ $\underline{v}_2 = \begin{bmatrix} -3 \\ 9 \\ -6 \end{bmatrix}$ $\underline{v}_3 = \begin{bmatrix} 5 \\ -7 \\ h \end{bmatrix}$ LI?

Let

$A = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}$. solve for $A\underset{\sim}{x} = \emptyset$

if A has 3 pivots $\Rightarrow v_1, v_2, v_3$ must be LI

$p = 3 \quad n = 3$

$$[A|0] = \left[\begin{array}{ccc|c} 1 & -3 & 5 & 0 \\ -3 & 9 & -7 & 0 \\ 2 & -6 & h & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} \underline{\underline{1}} & -3 & 5 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & h-10 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} \underline{\underline{1}} & -3 & 5 & 0 \\ 0 & 0 & \underline{8} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Got 2 pivots!

For every h x_2 is a free variable

$\therefore A\underset{\sim}{x} = \emptyset$ has a non-trivial solution.

$\Rightarrow \underset{\sim}{v_1}, \underset{\sim}{v_2}, \underset{\sim}{v_3}$ are LD //

For what value of h is $\underline{v_3}$ in Span$\{\underline{v_1}, \underline{v_2}\}$

Can we write v_3 as a linear combination of vectors v_1, v_2:

$\underset{\sim}{v_3} = B\underset{\sim}{x} = \begin{bmatrix} v_1 & v_2 \end{bmatrix} \underset{\sim}{x}$

$$\left[\begin{array}{cc|c} 1 & -3 & 5 \\ -3 & 9 & -7 \\ 2 & 6 & h \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -3 & 5 \\ 0 & 0 & 8 \\ 0 & 0 & h-10 \end{array}\right] \rightarrow \text{inconsistent}$$

$B \qquad v_3$

$\therefore$ V_3 not in span of $V_1, \underset{\sim}{V_2}$.

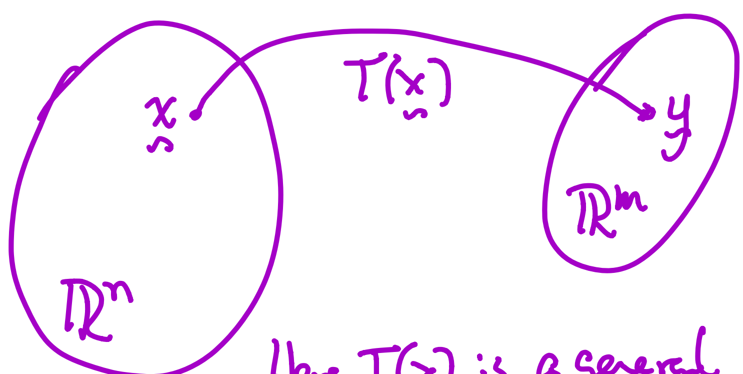
Cannot be written as linear combination of $\underset{\sim}{v_1}, \underset{\sim}{v_2}$

LINEAR TRANSFORMATIONS

A Transformation (or mapping or function)

written as $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$

for finite dimensional real spaces.



Here $T(x)$ is a general transformation.

Linear transformations of finite-dimensional vectors spaces accomplished via $T = A$

A is a matrix

General Case $\quad T: \mathbb{R}^n \to \mathbb{R}^m$

Domain of T

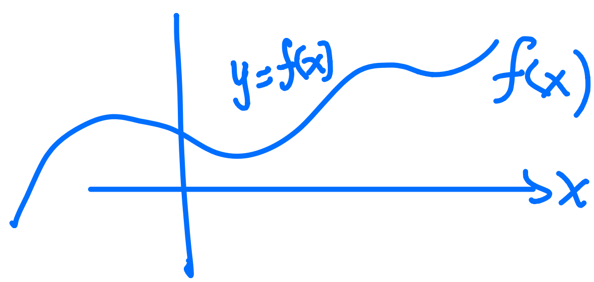
RANGE OF T (co-domain)

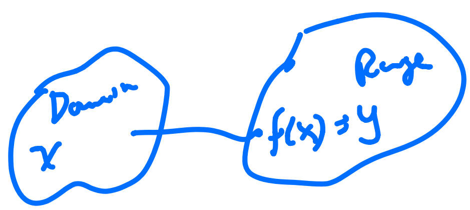
ex) $T = f(x)$

$f : x \to f(x)$

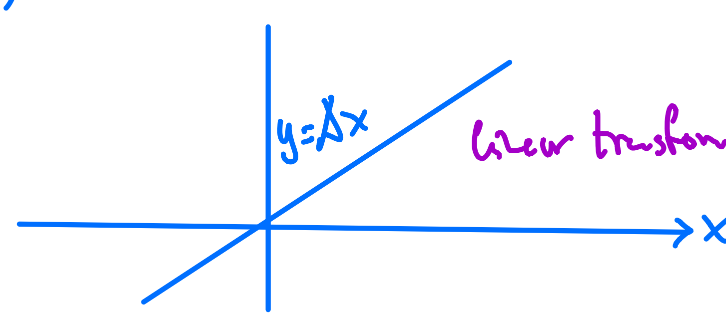
$x \in \mathbb{R}^1 \qquad f(x) \in \mathbb{R}^1$

Nonlinear Transformation





ex) $T: x \to Ax \qquad x \in \mathbb{R}^1$, A is a number



Linear transformation

$T: \mathbb{R}^n \to \mathbb{R}^m$ T is linear

$\Rightarrow$ T is a matrix.

$A \in \mathbb{R}^{m \times n}$ matrix

ex) $A\underset{\sim}{x} = y \in \mathbb{R}^m$

$x \in \mathbb{R}^n$

y is said to be the image of $\underset{\sim}{x}$ under the transformation of A.

ex) if $T(x) = A\underline{x} = \begin{bmatrix} x_1 - 3x_2 \\ 3x_1 + 5x_2 \\ -x_1 + 7x_2 \end{bmatrix} = y$

find A:

$A \in \mathbb{R}^{m \times n}$ $n = 2$

$m = 3$

$$y = \underset{\sim}{a_1} x_1 + \underset{\sim}{a_2} x_2 = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} x_1 + \begin{bmatrix} -3 \\ 5 \\ 7 \end{bmatrix} x_2$$

$$A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$$