COORDINATE SYSTEMS

Let $\mathbb{B} = \{\underset{\sim}{b}_1, \underset{\sim}{b}_2, \dots, \underset{\sim}{b}_n\}$ be a basis for a vectorspace V. Then $\exists\, \underset{\sim}{x} \in V$, unique s.t. $\underset{\sim}{x} = c_1 \underset{\sim}{b}_1 + c_2 \underset{\sim}{b}_2 + \dots c_n \underset{\sim}{b}_n$.

The COORDINATES of $\underset{\sim}{x}$ relative to the basis $\mathbb{B}$ are the c_i's $i = 1, 2, \dots, n$.

ex) $\mathbb{R}^2$ basis $\mathbb{B} = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\} = \{\hat{e}_1, \hat{e}_2\}$

so $\begin{bmatrix} 6 \\ 2 \end{bmatrix} = \underset{\sim}{x} = 6\hat{e}_1 + 2\hat{e}_2$

so the coordinates are $6, 2$.

ex) $\underset{\sim}{b}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\underset{\sim}{b}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

let $\underset{\sim}{x} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

Find the coordinates $[x]_{\mathbb{B}}$ relative to $\mathbb{B}$:

$$c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

find $[x]_{\mathbb{B}} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ //

We'll write $\underset{\sim}{x} = \sum_{i=1}^{n} c_i \underset{\sim}{b_i}$

$$\underset{\sim}{x} = P_{\mathbb{B}} [x]_{\mathbb{B}}$$

coordinates wrt $\mathbb{B}$

change of coordinates matrix

So $[x]_{\mathbb{B}} = P_{\mathbb{B}}^{-1} \underset{\sim}{x}$

$P_{\mathbb{B}}^{-1}$ exists & $P_{\mathbb{B}}$ is 1 to 1

Isomorphism: A coordinate transformation that is 1 to 1 in which two vector spaces (which are called isomorphic) are equivalent in every vector space operation.

So different named elements and notation but are the "same" in the vector space sense.

ex) $\mathbb{B} = \{\underbrace{1+t^2}_{b_1}, \underbrace{t+t^2}_{b_2}, \underbrace{1+2t+t^2}_{b_3}\}$

is a basis for $\mathbb{P}_2(t)$, the family of polynomials of degree at most 2.

$p \in \mathbb{P}_2(t) \qquad p(t) = a_0 t^0 + a_1 t^1 + a_2 t^2$

a_i are numbers.

Suppose $p(t)$ has a domain $t \in [-1, 1]$

then LI for the basis in $\mathbb{B}$

is given by $\int_{-1}^{1} b_i b_j w(t)\,dt = 0$ if $i \neq j$

$w(t)$ is a "weight" //

For $p(t) = 1+4t+7t^2$ find the coordinate vector relative to $\mathbb{B}$:

$$c_1(1+t^2) + c_2(t+t^2) + c_3(1+2t+t^2) = 1+4t+7t^2$$

$$(c_1+c_3)t^0 + (c_2+2c_3)t^1 + (c_1+c_2+c_3)t^2 = 1+4t+7t^2$$

$$\therefore \quad c_1 + c_3 = 1$$

$$c_2 + 2c_3 = 4$$

$$c_1 + c_2 + c_3 = 7$$

$$[x]_{\mathbb{B}} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ -1 \end{bmatrix}$$

//

ex) Let $\underset{\sim}{v}_1 = \begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix}$ $\underset{\sim}{v}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ $\underset{\sim}{x} = \begin{bmatrix} 3 \\ 12 \\ 7 \end{bmatrix}$

and $\mathbb{B} = \{ \underset{\sim}{v}_1, \underline{v}_2 \}$. Then $\mathbb{B}$ is the basis for $H = \text{span}\{ \underline{v}_1, \underline{v}_2 \}$. Determine if $\underset{\sim}{x}$ is in H and if it is, find the coordinate vector of x relative to $\mathbb{B}$:

if $\underset{\sim}{x} \in H$ then

$$c_1 \begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 12 \\ 7 \end{bmatrix}$$

show that the implied system is consistent.

$$\left[\begin{array}{cc|c} 3 & -1 & 3 \\ 6 & 0 & 12 \\ 2 & 1 & 7 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right]$$

$c_1 = 2$, $c_2 = 3$

$$[x]_{\mathbb{B}} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

ex) Find the change of coordinates matrix, from $\mathbb{B}$ to standard basis in $\mathbb{R}^2$, where

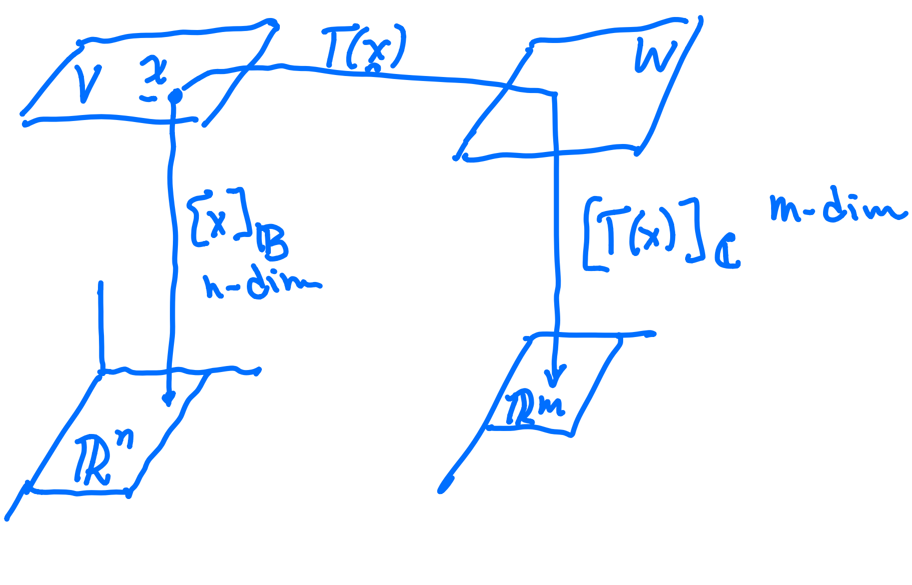
$$\mathbb{B} = \left\{ \begin{pmatrix} 1 \\ -3 \end{pmatrix}, \begin{pmatrix} 2 \\ -5 \end{pmatrix} \right\} = \{ \underset{\sim}{b}_1 \;\; \underset{\sim}{b}_2 \}$$

The standard basis is $\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$

$$P_{\mathbb{B}} = \begin{bmatrix} b_1 & b_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$$

Eigenvectors and Linear Transformations

V & W are vector spaces



V: $\mathbb{B}$ basis $= \{ b_1, b_2, \ldots, \underline{b}_n \}$

W: $\mathbb{C}$ basis

$\underset{\sim}{x} \in V \qquad \underset{\sim}{x} = r_1 \underline{b}_1 + r_2 \underline{b}_2 \cdots r_n \underline{b}_n$

$$[x]_{\mathbb{B}} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \end{bmatrix}$$

$$T(x) = T(r_1 b_1 + r_2 b_2 \cdots r_n b_n)$$

$$= r_1 T b_1 + r_2 T b_2 \cdots + r_n T b_n$$

The coordinate mapping of W to $\mathbb{R}^m$ is

$$[T(x)]_{\mathbb{C}} = r_1 [T(b_1)]_{\mathbb{C}} + r_2 [T b_2]_{\mathbb{C}} \cdots + r_n [T b_n]_{\mathbb{C}}$$

In matrix form

$$[T(x)]_{\mathbb{C}} = M [x]_{\mathbb{B}}$$

where $M = \left[[T b_1]_{\mathbb{C}} \; [T b_2]_{\mathbb{C}} \cdots [T b_n]_{\mathbb{C}} \right]$