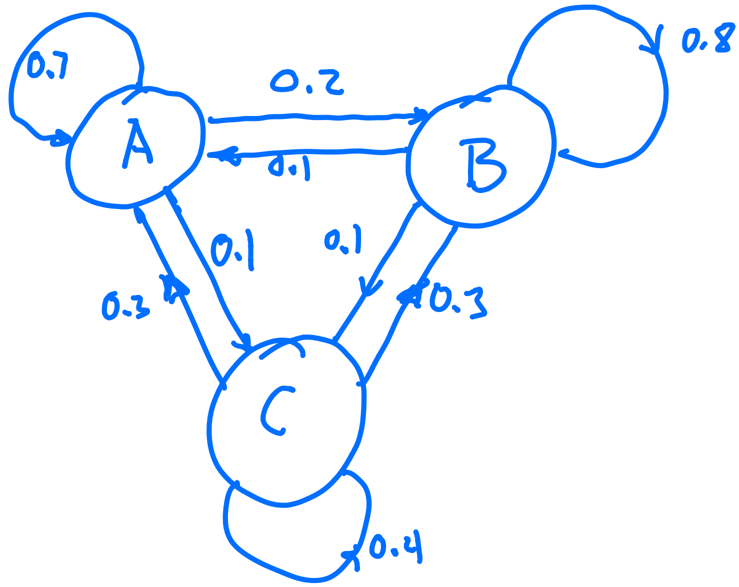
ex) VOTING



Suppose $x_0 = \begin{bmatrix} 0.55 \\ 0.40 \\ 0.05 \end{bmatrix}$ $\begin{matrix} A \\ B \\ C \end{matrix}$ might represent the percentage of voters in any of the 3 parties

Find $x_1 = P x_0$

$P = \begin{bmatrix} 0.7 & 0.1 & 0.3 \\ 0.2 & 0.8 & 0.3 \\ 0.1 & 0.1 & 0.4 \end{bmatrix}$ the way voters are found to vote (from experiment or from a tally of the vote, post-facto)

$x_1 = \begin{bmatrix} 0.440 \\ 0.445 \\ 0.115 \end{bmatrix}$ the outcome of the vote.

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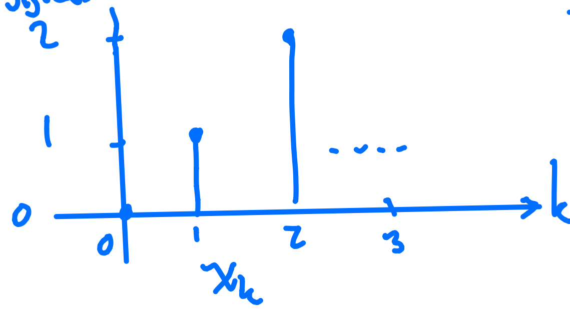
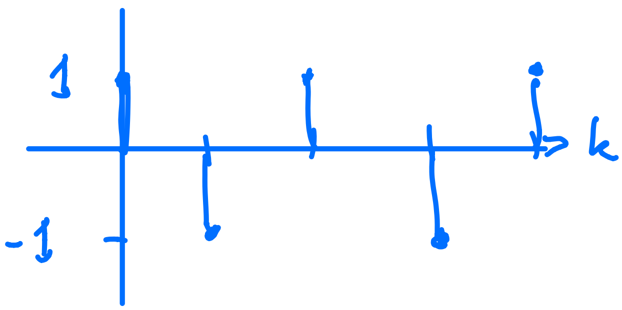
Discrete Signals: by relating difference equations and discrete signals, one can whether a signal has structure (rather than being totally random). One can also infer whether there might be a finite-dimensional space of in which the signal lives.

Discrete Signals:



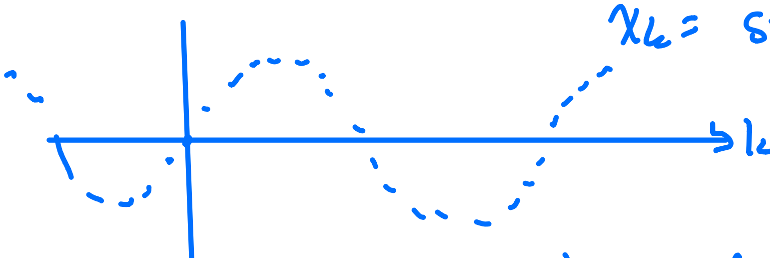
$x_k = k \qquad k = 0, 1 \ldots$

Degree of freedom (DOF) are 1



$x_k = (-1)^k \quad k = 0, 1 \ldots$

DOF 1



$x_k = \sin\left(\frac{2\pi}{N} k\right) \qquad k = 0, \pm 1 \ldots$

DOF is N

Determine the # of degrees of freedom in a time series... search for a LI basis

Suppose Given 3 sequences $\{u_k, v_k, w_k\}, k = 0, 1 \ldots$

want to determine whether they are

LI: see if $c_1 = c_2 = c_3 = 0$

$$c_1 u_k + c_2 v_k + c_3 w_k = 0$$

$\forall k.$ //

Take $\{u_k, w_k\}$ $k = 0, 1, \ldots n$

one way to show that $\{u_k, w_k\}$ are LI is to write

$$c_1 u_k + c_2 w_k = 0$$

$$c_1 u_{k+1} + c_2 w_{k+1} = 0$$

$$c_1 u_{k+2} + c_2 w_{k+2} = 0$$

$$\vdots$$

suffices to show that

$$\begin{bmatrix} u_k & w_k \\ u_{k+1} & w_{k+1} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = 0$$

if $\det \begin{bmatrix} u_k & w_k \\ u_{k+1} & w_{k+1} \end{bmatrix} \neq 0$ then

u_k, w_k LI $\forall k.$

ex) Show that $u_k = 1^k$, $w_k = (-2)^k$, $v_k = 3^k$ are LI

$$c_1 u_k + c_2 w_k + c_3 v_k = 0$$

with $c_1 = c_2 = c_3 = 0$

need 3 equations

$$C = \begin{bmatrix} 1^k & (-2)^k & 3^k \\ 1^{k+1} & (-2)^{k+1} & 3^{k+1} \\ 1^{k+2} & (-2)^{k+2} & 3^{k+2} \end{bmatrix}$$

Is called the Casorati Matrix

show that $\det C \neq 0$

suffices to pick any k, $k=0$, say:

$$C(k=0) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 3 \\ 1 & 4 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 2 \\ 0 & 0 & 10 \end{bmatrix}$$

3 pivots $\therefore$ LI

Rmk: If C is not invertible $\Rightarrow u_k, v_k, w_k$ may not be Linearly dependent. HOWEVER if they are all solution to the same homogeneous difference equation

$\Rightarrow$ either C is invertible $\forall k$ & signals are LI

OR C is not invertible $\forall k$ & signals are LD.

ex) $y_k = k^2$ and $z_k = 2k|k|$

are they LI?

$$C(k) = \begin{bmatrix} k^2 & 2k|k| \\ (k+1)^2 & 2(k+1)|k+1| \end{bmatrix}$$

$$C(0) = \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix} \quad C(-1) = \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix} \quad C(-2) = \begin{bmatrix} 4 & -8 \\ 1 & -2 \end{bmatrix}$$

$C(0)$ & $C(-1)$ are not invertible

$$\det C(k) = k^2 2(k+1)|k+1| - 2k|k|(k+1)^2$$

if $k=-1 \Rightarrow \det C_k = 0$

$k=0 \Rightarrow \det C_k = 0$

Generally: if $k < -1$ $(k+1) < 0 \Rightarrow \det C(k) = 0$

if $k > 0$ $(k+1) > 0 \Rightarrow \det C(k) = 0$

$\therefore$ $C(k)$ not invertible $\forall k$ $\therefore$ $\det C_k = 0$

cannot be used to determine whether y_k, z_k are LI

However, it is clear that

k^2 & $2k|k|$ are not multiples of each other $(\forall k)$.

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