SIMILARITY TRANSFORMATIONS

Th'm: Let A, B be $n \times n$ matrices. They are similar if they have the same characteristic polynomial.

For A the characteristic polynomial is $\det(A - \lambda I) = p(\lambda) = 0$

If A, B are similar then the decomposition

$$V^{-1} A V = B$$

is possible, where V is an invertible matrix.

$V^{-1} A V$ is the similarity transformation.

Pf: $B - \lambda I = V^{-1} A V - \lambda I$

$$= V^{-1} A V - \lambda V^{-1} V$$

$$B - \lambda I = V^{-1}(A - \lambda I)V$$

$$\det(B - \lambda I) = \det\left[V^{-1}(A - \lambda I)V\right]$$

$$\det(B - \lambda I) = \det(V^{-1})\det(A - \lambda I)\det V$$

but $\det(V^{-1})\det(V) = 1$

$$\therefore \det(B - \lambda I) = \det(A - \lambda I) \quad //$$

$\det(A - \lambda I) = \det(B - \lambda I)$ does not necessarily imply that the e'values of A and B are the same //

DIAGONALIZATION of a square matrix is an application of similarity transformations

Factor $A = V \Lambda V^{-1}$ ($)

Λ is a diagonal matrix

$A = V \Lambda V^{-1}$ A $n \times n$ matrix

$$\Lambda = \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix}$$

λ_i are the eigenvalues of A.

V is composed of columns corresponding to e'vectors of A.

Not every $n \times n$ matrix can be diagonalized:

If n LI eigenvectors cannot be found then A does not have a diagonalization.

$$\Lambda = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$$ for A $n \times n$ matrix

λ_i can be complex

If ($) is possible we say **A is diagonalizable**

The V is clearly made up of the e'vectors of A.

ex) $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$ Find $A = V\Lambda V^{-1}$
or $AV = V\Lambda$

1. Find e'values of A

$$p(\lambda) = (\lambda - 1)(\lambda + 2)^2 = 0$$

$$\lambda_1 = 1, \ \lambda_2 = -2, \ \lambda_3 = -2$$

2. Find e'vectors of A

$\lambda_1 = 1$

$$(1-1)x_1 + 3x_2 + 3x_3 = 0$$
$$-3x_1 - 4x_2 - 3x_3 = 0$$
$$3x_1 + 3x_2 + 0x_3 = 0$$

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$\lambda_2 = 2$

$$(1+2)x_1 + 3x_2 + 3x_3 = 0$$
$$-3x_1 - 3x_2 - 3x_3 = 0$$
$$3x_1 + 3x_2 + 0x_3 = 0$$

$$v_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \quad v_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

v_1, v_2, v_3 are LI

For $V = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$$\Lambda = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

check that $AV = V\Lambda$