HW8

SECTION 6.5

(1) Find the solution and draw the graph of solution to

$$y'' + 2y' + 2y = \delta(t-\pi)$$

$$y(0) = 1 \qquad y'(0) = 0$$

(2) Suppose $y'' + y = f(t)$

$$y(0) = 0 \qquad y'(0) = 0$$

where $f(t) = \sum_{k=1}^{\infty} \delta(t - k\pi)$

(a) Without solving the problem, try to predict the nature of the solution

(b) Test your prediction by finding the solution

(c) Determine what happens after the sequence of impulses ends.

Section 6.6

(3) Find the Laplace transform of

(a) $f(t) = \int_0^t e^{-(t-\tau)} \sin\tau \, d\tau$

(b) $f(t) = \int_0^t (t-\tau) e^{\tau} \, d\tau$

(4) Find the inverse Laplace transform

(a) $F(s) = \dfrac{1}{(s+1)^2(s^2+4)}$

(b) $F(s) = \dfrac{G(s)}{s^2+1}$

(5) Express the solution of the initial value problem

$y'' + 3y' + 2y = \cos at$

$y(0) = 1 \quad y'(0) = 0$

as a convolution integral.

(6) Solve the given Volterra integral equation by using Laplace transforms

$$\phi(t) + 2\int_0^t \cos(t-\tau)\,\phi(\tau)\,d\tau = e^{-t}$$

(7) Solve

$$\phi(t) - \frac{1}{2}\int_0^t (t-\tau)^2\,\phi(\tau)\,d\tau = -t$$

$$\phi(0) = 1$$