HW7

Section 6.1

① Sketch the graph of $f(t)$, determine whether $f(t)$ is continuous, piecewise continuous, or neither, on the interval $0 \le t \le 3$.

$$f(t) = \begin{cases} t^2 & 0 \le t \le 1 \\ 2+t & 1 < t \le 2 \\ 6-t & 2 < t \le 3 \end{cases}$$

② Recall that $\cos bt = \dfrac{e^{ibt} + e^{-ibt}}{2}$ and $\sin bt = \dfrac{e^{ibt} - e^{-ibt}}{2i}$.

Using these identities, find the Laplace transform of (a) $\cos bt$

(b) $e^{at} \sin bt$

assume a, b are real constants.

Section 6.2

(3) Find the inverse laplace transform of

(a) $F(s) = \dfrac{3}{s^2+4}$

(b) $F(s) = \dfrac{8s^2-4s+12}{s(s^2+4)}$

(4) Use the Laplace transform to solve

$$y'' - 2y' + 2y = 0 \; ; \quad y(0) = 0 \qquad y'(0) = 1$$

(5) Same as above.

$$y^{(4)} - 4y''' + 6y'' - 4y' + y = 0$$

$$y(0) = 0 \; ; \; y'(0) = 1 \; ; \; y''(0) = 0 \; ; \; y'''(0) = 1$$

(6) Find the Laplace transform $Y(s) = \mathcal{L}(y)$

of

$$y'' + 4y = \begin{cases} 1 & 0 < t < \pi \\ 0 & \pi \leq t < \infty \end{cases}$$

$$y(0) = 1 \qquad y'(0) = 0$$

section 6.3

(7) Find the Laplace transform of

$$f(t) = \begin{cases} 0 & t<0 \\ (t-2)^2 & t \geq 2 \end{cases}$$

(8) Find the Laplace transform of

$$f(t) = (t-3)\,u_2(t) - (t-2)\,u_3(t)$$

(9) Find the inverse Laplace transform of

$$F(s) = \frac{e^{-s} + e^{-2s} - e^{-3s} - e^{-4s}}{s}$$

section 6.4

(10) (a) Find solution to

$$y'' + y = \begin{cases} 1 & 0 \leq t < 3\pi \\ 0 & 3\pi \leq t < \infty \end{cases}$$

$y(0) = 0 \qquad y'(0) = 1$

(b) Plot solution & forcing. Explain how they are related.

(11) Find Solution, plot solution and forcing

$$y'' + 4y = \sin t - u_{2\pi}(t)\sin(t-2\pi)$$

$$y(0) = 0 \qquad y'(0) = 0$$

(12) A spring-mass system satisfies the IVP

$$u'' + \frac{1}{4}u' + u = k\,g(t)$$

$$u(0) = 0 \qquad u'(0) = 0$$

where $g(t) = u_{3/2}(t) - u_{5/2}(t)$

$k > 0$ is a parameter

(a) Sketch $g(t)$

(b) Solve initial value problem

(c) Plot solution for $k = \frac{1}{2}$, $k = 1$, $k = 2$.
Describe how the solution depends on k.

(d) Find, approximately, the smallest value of k for which a solution $u(t)$ reaches the value of 2.

(e) Suppose $k = 2$. Find the time τ after which $|u(t)| < 0.1$ for all $t > \tau$.

HW 8

SECTION 6.5

(13) Find the solution and draw the graph of solution to

$$y'' + 2y' + 2y = \delta(t - \pi)$$

$$y(0) = 1 \qquad y'(0) = 0$$

(14) Suppose $y'' + y = f(t)$

$$y(0) = 0 \qquad y'(0) = 0$$

where $f(t) = \sum_{k=1}^{\infty} \delta(t - k\pi)$

(a) Without solving the problem, try to predict the nature of the solution

(b) Test your prediction by finding the solution

(c) Determine what happens after the sequence of impulses ends.