HW6

SECTION 3.6

1. Use the method of variation of parameters to solve for a particular solution.

$$y'' - 5y' + 6y = 2e^t$$

then use the method of undetermined coefficients to determine if your answer is correct.

2. Find the general solution of

$$y'' + 9y = 9\sec^2 3t \quad , \quad 0 < t < \pi/6$$

3. Verify that $y_1(t) = t^2$ & $y_2(t) = t^{-1}$

are homogeneous solutions of

$$t^2 y'' - 2y = 3t^2 - 1 \quad t > 0$$

use these to find the particular solution of the ordinary differential equation.

4. Same question, with $y_1 = x^2$ $\quad y_2 = x^2 \ln x$

$$x^2 y'' - 3xy' + 4y = x^2 \ln x \quad , \quad x > 0$$

SECTION 3.7

(5) Given $u = 3\cos 2t + 4\sin 2t$

find R, ω_0, δ so that

$$u = R\cos(\omega_0 t - \delta)$$

(6) A series circuit has a capacitor $C = 0.25 \cdot 10^{-8}$ F and an inductor of 1 H.
If the initial charge on the capacitor is 10^{-6} C and there is no initial current find the charge Q in the capacitor at any later time.

(7) A mass ~~weight~~ weighing 8 lb stretches a spring 1.5 in. The mass is also attached to a damper with coefficient γ. Determine the value of γ for which the system is critically damped; be sure to give the units of γ

SECTION 3.8

(8) A mass of 5 Kg stretches a spring 10 cm. The mass is acted on by an external force of $10 \sin\left(\frac{t}{2}\right)$ Newtons. It moves in a

medium that imparts a viscous force of 2 Newtons when the speed of the mass is 4 cm/s

If the mass is set in motion from its equilibrium position with an initial velocity of 3 cm/s formulate an initial value problem describing the motion of the mass.

(9) (a) find the solution of problem (8)
(b) identify transient and steady parts of solution
(c) Plot the ~~state~~ steady state part
(d) If the given force is replaced by a force of $2 \cos \omega t$, find the frequency ω for which the amplitude of the forced response is maximum.

(10) Find the solution to the IVP

$$u'' + u = F(t) \ ; u(0) = 0, \ u'(0) = 0$$

where

$$F(t) = \begin{cases} F_0 t & 0 \le t < \pi \\ F_0(2\pi - t) & \pi \le t < 2\pi \\ 0 & t \geq 2\pi \end{cases}$$

Hint: Treat each time interval separately and match the solutions in the intervals by requiring that u and u' be continuous functions of t.