HW1

SECTION 1.1

① Draw a "direction" or "slope" field, for

$\frac{dy}{dt} = y' = 2y - 3$

Based on the field, determine the behavior of y as $t \to \infty$

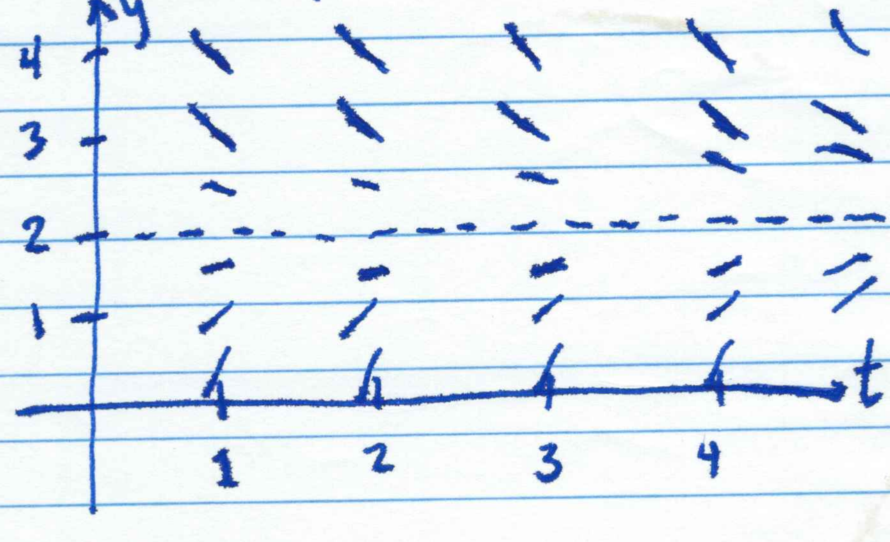
② Write down an ODE of the form

$\frac{dy}{dt} = ay + b$; a, b are constants

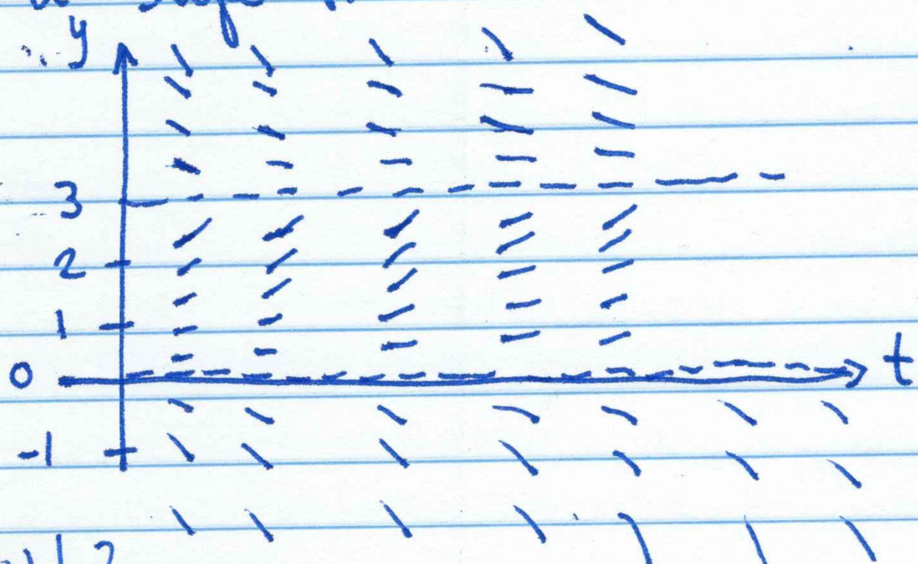
such that all solutions approach $y = 3$ as $t \to \infty$.

③ Draw slope field to $y' = y(4-y)$

④ Propose a reasonable first order equation that generates a slope field



(5) Propose a first order equation that generates a slope field



SECTION 1.2

(6) Solve
$$\begin{cases} \dfrac{dy}{dt} = -y+5 & t>0 \\ y(0) = y_0, \text{ a constant} \end{cases}$$

(7) Consider the ODE $\dfrac{dy}{dt} = ay - b$

(a) Find the equilibrium solution y_e

(b) Let $Y = y - y_e$. This $Y(t)$ is the deviation from the equilibrium solution. Find the differential equation satisfied by $Y(t)$

⑧ A mouse population satisfies

$$\frac{dp}{dt} = 0.5p - 450 \quad , t > 0$$

$p = p(t)$ is the population at time t (years)

a) Find the time at which the population becomes extinct if $p(0) = 850$

b) Find the time of extinction if $p(0) = p_0$ where $0 < p_0 < 900$

c) Find the initial population p_0 if the population is to become extinct in 1 year.

⑨ Suppose a building loses heat in accordance with Newton's law of cooling

$$\cancel{\begin{cases} \dfrac{dT}{dt} = k(T - T_0) & t > 0 \\ T(0) = T_I \end{cases}}$$

$$\begin{cases} \dfrac{dT}{dt} = k(T - T_0) & , t > 0 \\ T(0) = T_I \end{cases}$$

$k = 0.15$ 1/hour , t is in hours

T is in °F

Assume interior building temperature is 70°F and at $t=0$ it loses its heating system. If the outdoor temperature is 10°F, how long will it take for the interior temperature to reach 32°F?

SECTION 1.3

(10) Determine whether the equation

$$t^2 \frac{d^2y}{dt^2} + t\frac{dy}{dt} + 2y = \sin t$$

is linear or nonlinear and what order it is.

(11) Same questions, for

$$\frac{d^4y}{dt^4} + \frac{d^3y}{dt^3} + \frac{d^2y}{dt^2} + \frac{dy}{dt} + y = 1$$

(12) Same questions, for

$$\frac{d^2y}{dt^2} + t\frac{dy}{dt} + (\cos^2 t)y = t^3$$

(13) Determine the value of r in $y = e^{rt}$

if y satisfies $y'' - y = 0$

(14) Determine the value of β in $y = t^{\beta}$ if

y satisfies $t^2 y'' + 4ty' + 2y = 0$, $t > 0$

(15) Determine the order of the partial differential equation

$$u_t + uu_x = 1 + u_{xx}$$

where $u = u(x,t)$. Determine whether linear or nonlinear.

SECTION 2.1

(16) Determine the solution of $y' + 3y = t + e^{-2t}$

and how the solution behaves as $t \to \infty$

(17) Solve the initial value problem

$$\begin{cases} y' - y = 2te^{2t} \\ y(0) = 1 \end{cases}$$

(18) Solve $\begin{cases} ty' + (t+1)y = t & , t > 0 \\ y(\ln 2) = 1 \end{cases}$

(19) Show that all solutions of $2y' + ty = 2$ approach a limit as $t \to \infty$. Find the limiting value. See EXAMPLE 4 in Section 2.1

The solution is

$$y(t) = e^{-t^4/4} \int_0^t e^{s^2/4}\, ds + c e^{-t^2/4}$$

Hint: Use L'Hopital's Rule