Math 256, Winter 2014
Scarborough
Exam 2

NAME (print) ______________________________

NAME (sign) ______________________________

Student Number ____________________________

Class Time (Circle One) 9:00 (section 050) 1:00 (section 020)

Recitation Instructor (Circle One) Thomas Morrill Dwight Holland

Recitation Time (Circle One) 8:00 9:00 10:00 11:00 1:00 2:00 3:00 4:00

This exam is closed book. Calculators and notes are not allowed.

This exam has 10 pages with 14 problems. You should scan all the problems before you begin the test so you can plan out your time. The time limit on this exam is 1 hour 20 minutes. Remember multiple choice does not necessarily mean quick answer.

Please record your answers to problems 1 through 11 on the provided Scantron sheet using a **#2 pencil**. Do all work on the exam sheet. **No scratch paper is allowed.** Also record your answer on these pages by circling the answer. **The answer on your Scantron sheet is "your final answer!"**

Problem 1 is worth 10 points
Problems 2 – 11 are worth 6 points each.
Problems 12, 13, and 14 are worth 10 points each.

Total possible points $(10)(1)+(6)(10)+(3)(10)=100$ points

1. I have properly put and bubbled my name and student number on the Scantron form.
I have legibly printed and signed my name in the proper spaces above.
I have legibly printed my student number. in the proper space above.
I have circled my class, recitation instructor, and recitation time above.

(**A**) TRUE

$y'' + p(t)y' + q(t)y = g(t) \qquad y = c_1y_1 + c_2y_2 + y_p$

$y_2 = u\ y_1 \qquad y_1u'' + (2y_1' + py_1)u' = 0$

$y_p = u_1y_1 + u_2y_2,\quad u_1' = \frac{W_1g}{W},\quad u_2' = \frac{W_2g}{W},\quad W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix},\ W_1 = \begin{vmatrix} 0 & y_2 \\ 1 & y_2' \end{vmatrix},\ W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & 1 \end{vmatrix}$

$y''' + p(t)y'' + q(t)y' + r(t)y = g(t) \qquad y = c_1y_1 + c_2y_2 + c_3y_3 + y_p$

$y_p = u_1y_1 + u_2y_2 + u_3y_3 \qquad u_1' = \frac{W_1g}{W},\quad u_2' = \frac{W_2g}{W},\quad u_3' = \frac{W_3g}{W},$

$W = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix},\ W_1 = \begin{vmatrix} 0 & y_2 & y_3 \\ 0 & y_2' & y_3' \\ 1 & y_2'' & y_3'' \end{vmatrix},\ W_2 = \begin{vmatrix} y_1 & 0 & y_3 \\ y_1' & 0 & y_3' \\ y_1'' & 1 & y_3'' \end{vmatrix},\ W_3 = \begin{vmatrix} y_1 & y_2 & 0 \\ y_1' & y_2' & 0 \\ y_1'' & y_2'' & 1 \end{vmatrix}$

$\int \sin(x)\ dx = -\cos(x) + C$, $\int \cos(x)\ dx = \sin(x) + C$, $\int \tan(x)dx = \ln|\sec(x)| + C$,
$\int \cot(x)\ dx = -\ln|\csc(x)| + C$

$\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + C$, $\int \csc(x)dx = \ln|\csc(x) - \cot(x)| + C$, $\int \sec(x)\tan(x)dx = \sec(x) + C$,

$\int \csc(x)\cot(x)dx = -\csc(x) + C$, $\int \sec^2(x)dx = \tan(x) + C$, $\int \csc^2(x)dx = -\cot(x) + C$

$\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + C$

$\int \csc^3(x)dx = -\frac{1}{2}\csc(x)\cot(x) + \frac{1}{2}\ln|\csc(x) - \cot(x)| + C$

$\int \sin^2(x)dx = \frac{1}{2}x - \frac{1}{2}\sin(x)\cos(x) + C$, $\qquad \int \cos^2(x)dx = \frac{1}{2}x + \frac{1}{2}\sin(x)\cos(x) + C$

$\int x\ \sin(x)\ dx = \sin(x) - x\ \cos(x) + C$, $\qquad \int x\ \cos(x)\ dx = \cos(x) + x\ \sin(x) + C$

$\int e^{ax}\ \sin(bx)\ dx = \frac{e^{ax}\ (a\ \sin(bx) - b\ \cos(bx))}{a^2 + b^2} + C$, $\int e^{ax}\ \cos(bx)\ dx = \frac{e^{ax}\ (a\ \cos(bx) + b\ \sin(bx))}{a^2 + b^2} + C$

$\int \ln(x)\ dx = x\ln(x) - x + C$, $\qquad \int x\ e^{ax}\,dx = \frac{1}{a^2}(ax - 1)\ e^{ax} + C$

MAKE SURE YOU ANSWERED PROBLEM 1 from page 1

2. Find the general solution to $y'' - 4y' + 4y = 0$.

(**A**) $y = Ae^{-2t} + Be^{2t}$

(**B**) $y = Ae^{-2t} + Bte^{-2t}$

(**C**) $y = Ae^{2t} + Bte^{2t}$

(**D**) $y = e^{-2t}\left[A\cos(2t) + B\sin(2t)\right]$

(**E**) $y = e^{-4t}\left[A\cos(4t) + B\sin(4t)\right]$

3. Which of the following is a particular solution to $y'' - 2y' + y = 4\sin t$

(**A**) $y_p = 2\sin t$

(**B**) $y_p = 2\cos t$

(**C**) $y_p = 2\cos t + 4\sin t$

(**D**) $y_p = 2\cos t + 2\sin t$

(**E**) $y_p = 4\cos t - 2\sin t$

4. Find a particular solution to $y'' + 3y' - 4y = e^{2t}$

(**A**) $y_p = \frac{1}{6}e^{2t}$ (**B**) $y_p = \frac{1}{4}e^{2t}$ (**C**) $y_p = \frac{1}{3}e^{2t}$

(**D**) $y_p = \frac{1}{3}te^{2t}$ (**E**) $y_p = \frac{1}{9}te^{2t}$

5. Find a particular solution to $y'' + 2y' - y = 6$

(**A**) 0 (**B**) 3 (**C**) 6 (**D**) –6 (**E**) –3

6. Two solutions to $y'' + p(t)y' + q(t)y = 0$ are $y_1 = t$ and $y_2 = t^3$ where $p(t)$ and $q(t)$ are continuous on $(0,\infty)$. Find a particular solution to $y'' + p(t)y' + q(t)y = t$

(**A**) $y_p = \frac{1}{2}t^2$

(**B**) $y_p = \frac{1}{2}t^3$

(**C**) $y_p = \frac{1}{2}t^4$

(**D**) $y_p = \frac{1}{2}t\ln(t)$

(**E**) $y_p = \frac{1}{2}t^3\ln(t)$

7. For what value of λ is the system described by $y'' + 2\lambda y' + 9y = 0$ critically damped?

(**A**) 0

(**B**) $\frac{3}{2}$

(**C**) $\frac{9}{2}$

(**D**) 3

(**E**) 9

8. For what value of k does the system described by the differential equation $y'' + 16y = 3\cos(kt)$ exhibit resonance?

(**A**) 16 (**B**) 4 (**C**) 3 (**D**) 2

(**E**) 0

9. Determine the best form for y_p if the method of undetermined coefficients is to be used to solve the following differential equation. (Best means that all essential terms that are needed for the method are present and that there are no extra terms)

$$y''' + y'' = t^3 + \sin t$$

(**A**) $y_p = At^3 + B\sin t$

(**B**) $y_p = At^5 + B\sin t$

(**C**) $y_p = At^5 + B\cos t + C\sin t$

(**D**) $y_p = At^5 + Bt^4 + Ct^3 + Dt^2 + E\cos t + F\sin t$

(**E**) $y_p = At^5 + Bt^4 + Ct^3 + Dt^2 + Et + F + G\ t\ \cos t + H\ t\ \sin t$

10. Find the value of $\Gamma(5)$.

(**A**) $\sqrt{\pi}$ (**B**) 4 (**C**) 5 (**D**) 24

(**E**) 70

11. Recall that the Laplace transform of a function f is defined as $\mathcal{L}(f(t)) = \int_0^\infty f(t)e^{-st}\,dt$.

$$f(t) = \begin{cases} 0, & 0 \le t < 1 \\ 1, & 1 \le t < \infty \end{cases}$$ Find $\mathcal{L}(f(t))$.

(**A**) $\mathcal{L}(f(t)) = \begin{cases} 0, & 0 \le s < 1 \\ \dfrac{1}{s}, & 1 \le s < \infty \end{cases}, \; s > 0$

(**B**) $\mathcal{L}(f(t)) = \begin{cases} 0, & 0 \le s < 1 \\ \dfrac{e^{-s}}{s}, & 1 \le s < \infty \end{cases}, \; s > 0$

(**C**) $\mathcal{L}(f(t)) = \dfrac{e^{-s}}{s}, \; s > 0$

(**D**) $\mathcal{L}(f(t)) = e^{-s}, \; s > 0$

(**E**) $\mathcal{L}(f(t))$ does not exist.

For problems 12, 13, and 14 for full credit show all work.
Grading will be based on correctness, completeness, neatness, style and organization.

Place your final answer in the provided rectangle.
If your final answer is not in the rectangle you will not receive full credit.

12. One solution to $y'' - \frac{6}{t}y' + q(t)y = 0$ is $y_1 = t^2$ where $q(t)$ is continuous on $(0,\infty)$. Find a second linearly independent solution. HINT: The needed formula is on your formula sheet.

13. Find the general solution to $y'' + y = \sec^2(t)$

14. Find the solution to the initial value problem

$$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 12e^{-t}\ , \qquad y(0) = 0,\quad y'(0) = 0$$