Math 256, Spring 2015
Exam 2
KEY

NAME (print)____KEY ________________________________

NAME (sign) ____

Student Number____

Class Time (Circle One) 10:00 11:00 1:00 2:00

Recitation Time (Circle One)

8:00	9:00	10:00	11:00	12:00
1:00	2:00	3:00	4:00	

This exam is closed book, closed notes.
No Calculators allowed or needed
The formula sheet is on the last page.

This exam has 9 pages with 15 problems. You should scan all the problems before you begin the test so you can plan out your time. The time limit on this exam is 1 hour 20 minutes. Remember, multiple choice does not necessarily mean quick answer.

Please record your answers to problems 1 through 12 on the provided Scantron sheet using a **#2 pencil**. Do all work on the exam sheet. **No scratch paper is allowed.** No partial credit will be given for the multiple choice problems. **The answer on the Scantron sheet is your final answer.**

Problems 1 - 12 are worth 6 points each
Problem 13 is worth 8 points
Problems 14 and 15 are worth 10 points each

Total points possible: (12)(6) + (1)(8) + (2)(10) = 100

1. What is the largest interval on which the following initial value problem is guaranteed to have a unique solution:

$$(t+5)(t-7)\ y'' + 6t\ y' - 8y = 4\ln|t|$$
$$y(1) = 3$$
$$y'(1) = 0$$

(**A**) $(-5,7)$ (**B**) $(-\infty,\infty)$ (**C**) $(0,5)$ (**D**) $(0,7)$ (**E**) $(1,7)$

2. Find the general solution to $y'' - 4y' + 8y = 0$

(**A**) $y = Ae^{-6t} + Be^{2t}$ (**B**) $y = Ae^{-4t} + Be^{8t}$

(**C**) $y = e^{2t}\left[A\cos(2t) + B\sin(2t)\right]$ (**D**) $y = e^{-2t}\left[A\cos(4t) + B\sin(4t)\right]$

(**E**) $y = Ae^{2t} + Bt\ e^{2t}$

3. Find the general solution to $4y'' + 4y' + y = 0$

(**A**) $y = Ae^{-\frac{1}{2}t} + Bt\ e^{-\frac{1}{2}t}$ (**B**) $y = Ae^{-\frac{1}{2}t} + Be^{\frac{1}{2}t}$

(**C**) $y = e^{\frac{1}{2}t}\left[A\cos\left(\frac{t}{2}\right) + B\sin\left(\frac{t}{2}\right)\right]$ (**D**) $y = e^{-\frac{1}{2}t}\left[A\cos\left(\frac{t}{2}\right) + B\sin\left(\frac{t}{2}\right)\right]$

(**E**) $y = Ae^{4t} + Bt\ e^{4t}$

4. Find the general solution to $x^2 y'' + 3x\ y' + 5y = 0$

(**A**) $y = x^{-\frac{3}{2}}\left[A\cos\left(\frac{\sqrt{11}}{2}\ln|x|\right) + B\sin\left(\frac{\sqrt{11}}{2}\ln|x|\right)\right]$

(**B**) $y = x^{-\frac{3}{2}}\left[A\cos\left(\ln\left|\frac{\sqrt{11}}{2}x\right|\right) + B\sin\left(\ln\left|\frac{\sqrt{11}}{2}x\right|\right)\right]$

(**C**) $y = x^{-1}\left[A\cos(2\ln|x|) + B\sin(2\ln|x|)\right]$

(**D**) $y = Ax^{-1} + Bx^5$

(**E**) $y = Ax + Bx^5$

5. For what value of λ is the system described by $y'' + k\ y' + 16y = 0$ critically damped?

(**A**) 0 (**B**) 2 (**C**) 4 (**D**) 8 (**E**) 16

6. For what value of ω does the system described by $y'' + 9y = 4\cos(\omega t)$ exhibit resonance?

(**A**) 0 (**B**) 3 (**C**) 4 (**D**) 9 (**E**) 81

7. Find a particular solution to $y'' + 2y' + y = t$

(**A**) $y_p = \frac{1}{2}t$ (**B**) $y_p = 2t - 1$

(**C**) $y_p = 2t - 2$ (**D**) $y_p = t$

(**E**) $y_p = t - 2$

8. $e^{2\pi i}$ equals

(**A**) 0 (**B**) 1 (**C**) i (**D**) -1 (**E**) $-i$

9. Which of the following is the best choice for the form of the particular solution when using the Method of Undetermined Coefficients to solve the differential equation

$$y'' - 2y' + y = e^t$$

Note: Best means that all terms required by this method are included and no extra terms are included.

(**A**) $y = A\, e^t$

(**B**) $y = A\, t\, e^t$

(**C**) $y = A\, t^2\, e^t$

(**D**) $y = A\, t\, e^t + B\, e^t$

(**E**) $y = A\, t^2\, e^t + B\, t\, e^t + C\, e^t$

10. Find the solution to the initial value problem:

$$y'' + 3y' + 2y = 0, \qquad y(0) = 0,\ y'(0) = -2$$

(**A**) $y = 2e^{-2t} - 2e^{-t}$

(**B**) $y = e^{-2t} - e^{-t}$

(**C**) $y = \frac{9}{2}\sin\left(\frac{\sqrt{3}}{2}\, t\right)$

(**D**) $y = 2\, e^{-2t}\sin(3t)$

(**E**) $y = 2\, e^t - 2\, e^{2t}$

11. Find the solution to the initial value problem:

$$y'' + y = 2e^t, \quad y(0) = 0,\ y'(0) = 0$$

(**A**) $y = 0$

(**B**) $y = e^t$

(**C**) $y = 2\sin(t) - \cos(t) + e^t$

(**D**) $y = \sin(t) - 2\cos(t) + e^t$

(**E**) $y = e^t - \cos(t) - \sin(t)$

12. Use the Wronskian to determine which one of the following pairs of functions do **not** form a set of fundamental solutions to the differential equation $y'' - y = 0$.

(**A**) $\{e^t,\ e^{-t}\}$

(**B**) $\{e^t + e^{-t},\ e^t - e^{-t}\}$

(**C**) $\{e^t,\ e^t + e^{-t}\}$

(**D**) $\{e^t - e^{-t},\ e^{-t} - e^t\}$

(**E**) All of these form a set of fundamental solutions to $y'' - y = 0$.

For full credit show all work. No imaginary numbers are allowed in your final answer.
If you need more space please use the back of the previous page.

Place your final answer in the provided rectangle.
If your final answer is not in the rectangle you will not receive full credit.

13. Find the general solution to $y'' + 2y' + 5y = 3e^{-t}$

First find the general solution to the homogeneous equation $y'' + 2y' + 5y = 0$
The characteristic equation is

$$m^2 + 2m + 5 = 0$$

$$m = \frac{-2 \pm \sqrt{4-20}}{2} = -1 \pm 2i$$

$$y_h = e^{-t}\left[A\cos(2t) + B\sin(2t)\right]$$

Next use the method of undetermined coefficients to find a particular solution:

$$y_p = Ce^{-t}$$

$$y_p' = -Ce^{-t}$$

$$y_p'' = Ce^{-t}$$

Substitute into the differential equation:

$$Ce^{-t} - 2Ce^{-t} + 5Ce^{-t} = 3e^{-t}$$

$$4C = 3$$

$$C = \frac{3}{4}$$

$$y_p = \frac{3}{4}e^{-t}$$

$$y = y_h + y_p$$

$$y = e^{-t}\left[A\cos(2t) + B\sin(2t)\right] + \frac{3}{4}e^{-t}$$

$$y = e^{-t}\left[A\cos(2t) + B\sin(2t)\right] + \frac{3}{4}e^{-t}$$

For full credit show all work. . No imaginary numbers are allowed in your final answer.
If you need more space please use the back of the previous page.

Place your final answer in the provided rectangle.
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14. Find a second solution y_2 for which $W[y_1, y_2] \neq 0$ to

$t\,y'' - 2(t+1)\,y' + (t+2)\,y = 0$ given that $y_1(t) = e^t$ is a known solution.

We will use reduction of order to find a second solution:

Assume $y_2 = uy_1 = ue^t$

$y_2' = u'e^t + ue^t$

$y'' = u''e^t + u'e^t + u'e^t + ue^t = u''e^t + 2u'e^t + ue^t$

Substitue into the differential equation:

$t\left(u''e^t + 2u'e^t + ue^t\right) - 2(t+1)\left(u'e^t + ue^t\right) + (t+2)ue^t = 0$

$tu'' + 2tu' + tu - 2tu' - 2u' - 2tu - 2u + tu + 2u = 0$

$tu'' - 2u' = 0$

Let $w = u'$

$tw' - 2w = 0 \quad \Rightarrow \quad \dfrac{w'}{w} = \dfrac{2}{t} \quad \Rightarrow \quad \ln|w| = 2\ln|t| + C \quad \Rightarrow$ Take $w = t^2$

$u' = t^2$

$u = \dfrac{1}{3}t^3$

Then $y_2 = uy_1 = \dfrac{1}{3}t^3e^t$

Since any constant times a solution is a solution we can take $y_2 = t^3e^t$

$y_2 = t^3e^t$

For full credit show all work. . No imaginary numbers are allowed in your final answer.
If you need more space please use the back of the previous page.
Place your final answer in the provided rectangle.
If your final answer is not in the rectangle you will not receive full credit.

15. Find the general solution to $y'' + y = 2\sec(t)$

First find the solution to the homogeneous equation $y'' + y = 0$ $y_h = A\cos(t) + B\sin(t)$
$y_1 = \cos(t), \quad y_2 = \sin(t)$

We will use variation of parameters to find a particular solution:
$y_p = u_1 y_1 + u_2 y_2$

$$W\left[\cos(t),\sin(t)\right] = \begin{vmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{vmatrix} = \cos^2(t) + \sin^2(t) = 1$$

$$u_1' = \frac{2\sec(t)\begin{vmatrix} 0 & \sin(t) \\ 1 & \cos(t) \end{vmatrix}}{W} = 2\sec(t)\left(-\sin(t)\right) = -2\tan(t)$$

$$u_1 = \int -2\tan(t)\,dt = -2\ln\left|\sec(t)\right| = 2\ln\left|\cos(t)\right|$$

$$u_2' = \frac{2\sec(t)\begin{vmatrix} \cos(t) & 0 \\ -\sin(t) & 1 \end{vmatrix}}{W} = 2\sec(t)\cos(t) = 2$$

$$u_2 = \int 2\,dt = 2t$$

$$y_p = u_1 y_1 + u_2 y_2 = \cos(t)2\ln\left|\cos(t)\right| + 2t\sin(t)$$

$$y = y_h + y_p$$

$$y = A\cos(t) + B\sin(t) + \cos(t)2\ln\left|\cos(t)\right| + 2t\sin(t)$$

$$y = A\cos(t) + B\sin(t) + \cos(t)2\ln\left|\cos(t)\right| + 2t\sin(t)$$

$$y'' + p(t)y' + q(t)y = g(t) \qquad y = c_1y_1 + c_2y_2 + y_p$$

$$y_2 = u\ y_1 \qquad y_1u'' + (2y_1' + py_1)u' = 0$$

$$y_p = u_1y_1 + u_2y_2, \quad u_1' = \frac{W_1g}{W}, \quad u_2' = \frac{W_2g}{W}, \quad W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}, \; W_1 = \begin{vmatrix} 0 & y_2 \\ 1 & y_2' \end{vmatrix}, W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & 1_2 \end{vmatrix}$$

$$y''' + p(t)y'' + q(t)y' + r(t)y = g(t) \qquad y = c_1y_1 + c_2y_2 + c_3y_3 + y_p$$

$$y_p = u_1y_1 + u_2y_2 + u_3y_3 \qquad u_1' = \frac{W_1g}{W}, \quad u_2' = \frac{W_2g}{W}, \quad u_3' = \frac{W_3g}{W},$$

$$W = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix}, \quad W_1 = \begin{vmatrix} 0 & y_2 & y_3 \\ 0 & y_2' & y_3' \\ 1 & y_2'' & y_3'' \end{vmatrix}, \quad W_2 = \begin{vmatrix} y_1 & 0 & y_3 \\ y_1' & 0 & y_3' \\ y_1'' & 1 & y_3'' \end{vmatrix}, \quad W_3 = \begin{vmatrix} y_1 & y_2 & 0 \\ y_1' & y_2' & 0 \\ y_1'' & y_2'' & 1 \end{vmatrix}$$

$$\int \sin(x)\ dx = -\cos(x) + C, \qquad \int \cos(x)\ dx = \sin(x) + C$$

$$\int \tan(x)dx = \ln|\sec(x)| + C, \qquad \int \cot(x)\ dx = -\ln|\csc(x)| + C$$

$$\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + C, \int \csc(x)dx = \ln|\csc(x) - \cot(x)| + C, \int \sec(x)\tan(x)dx = \sec(x) + C,$$

$$\int \csc(x)\cot(x)dx = -\csc(x) + C, \int \sec^2(x)dx = \tan(x) + C, \int \csc^2(x)dx = -\cot(x) + C$$

$$\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + C$$

$$\int \csc^3(x)dx = -\frac{1}{2}\csc(x)\cot(x) + \frac{1}{2}\ln|\csc(x) - \cot(x)| + C$$

$$\int \sin^2(x)dx = \frac{1}{2}x - \frac{1}{2}\sin(x)\cos(x) + C, \qquad \int \cos^2(x)dx = \frac{1}{2}x + \frac{1}{2}\sin(x)\cos(x) + C$$

$$\int x\ \sin(x)\ dx = \sin(x) - x\ \cos(x) + C, \qquad \int x\ \cos(x)\ dx = \cos(x) + x\ \sin(x) + C$$

$$\int e^{ax}\ \sin(bx)\ dx = \frac{e^{ax}\left(a\ \sin(bx) - b\ \cos(bx)\right)}{a^2 + b^2} + C, \ \int e^{ax}\ \cos(bx)\ dx = \frac{e^{ax}\left(a\ \cos(bx) + b\ \sin(bx)\right)}{a^2 + b^2} + C$$

$$\int \ln(x)\ dx = x\ln(x) - x + C, \qquad \int x\ e^{ax}\,dx = \frac{1}{a^2}(ax - 1)\ e^{ax} + C$$