Mth 256, Exam 1
Scarborough
Spring 2011
Form 1

NAME (print) ______________________________

NAME (sign) ______________________________

Student Number ______________________________

Class Time (Circle One) 10:00 1:00

Recitation Instructor (Circle One) Karlan Wolfkill Noella Grady

Recitation Time (Circle One) 8:00 9:00 10:00 11:00 1:00 2:00 3:00 4:00

This exam is closed book, closed notes. A formula sheet is on page 2.
No Calculators allowed or needed

This exam has 8 pages with 15 problems. You should scan all the problems before you begin the test so you can plan out your time. The time limit on this exam is 50 minutes. Remember, multiple choice does not necessarily mean quick answer.

Please record your answers to problems 1 through 13 on the provided Scantron sheet using a **#2 pencil**. Do all work on the exam sheet. **No scratch paper is allowed.** No partial credit will be given for the multiple choice problems. **The answer on the Scantron sheet is your final answer.**

Mark and bubble the form number of the exam on the front of the Scantron sheet.
This exam is **Form 1**.

Problem 1 is worth 8 points
Problems 2 – 13 are worth 6 points each.
Problems 14 and 15 are worth 10 points each.

Total points possible: 8 +(12)(6)+(2)(10)=100

For problems 14 and 15 you must show all work for full credit.

1. I have properly recorded the form number of this exam on the Scantron sheet.
   This exam is **Form 1.**
   I have properly put and bubbled my name and student number on the Scantron form.
   I have legibly printed and signed my name in the proper spaces above.
   I have legibly printed my student number. in the proper space above.
   I have circled my recitation instructor's name, class time, and recitation time above.

   (**A**) TRUE

$$y'' + p(t)y' + q(t)y = g(t) \qquad y = c_1y_1 + c_2y_2 + y_p$$

$$y_2 = u\ y_1 \qquad y_1u'' + (2y_1' + py_1)u' = 0$$

$$y_p = u_1y_1 + u_2y_2\,, \quad u_1' = \frac{W_1g}{W}, \quad u_2' = \frac{W_2g}{W}, \quad W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}, \ W_1 = \begin{vmatrix} 0 & y_2 \\ 1 & y_2' \end{vmatrix}, W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & 1 \end{vmatrix}$$

$$y''' + p(t)y'' + q(t)y' + r(t)y = g(t) \qquad y = c_1y_1 + c_2y_2 + c_3y_3 + y_p$$

$$y_p = u_1y_1 + u_2y_2 + u_3y_3 \qquad u_1' = \frac{W_1g}{W}, \quad u_2' = \frac{W_2g}{W}, \quad u_3' = \frac{W_3g}{W},$$

$$W = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix}, \quad W_1 = \begin{vmatrix} 0 & y_2 & y_3 \\ 0 & y_2' & y_3' \\ 1 & y_2'' & y_3'' \end{vmatrix}, \quad W_2 = \begin{vmatrix} y_1 & 0 & y_3 \\ y_1' & 0 & y_3' \\ y_1'' & 1 & y_3'' \end{vmatrix}, \quad W_3 = \begin{vmatrix} y_1 & y_2 & 0 \\ y_1' & y_2' & 0 \\ y_1'' & y_2'' & 1 \end{vmatrix}$$

$\int \sin(x)\ dx = -\cos(x) + C$, $\int \cos(x)\ dx = \sin(x) + C$, $\int \tan(x)dx = \ln|\sec(x)| + C$,
$\int \cot(x)\ dx = -\ln|\csc(x)| + C$

$\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + C$, $\int \csc(x)dx = \ln|\csc(x) - \cot(x)| + C$, $\int \sec(x)\tan(x)dx = \sec(x) + C$,

$\int \csc(x)\cot(x)dx = -\csc(x) + C$, $\int \sec^2(x)dx = \tan(x) + C$, $\int \csc^2(x)dx = -\cot(x) + C$

$$\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + C$$

$$\int \csc^3(x)dx = -\frac{1}{2}\csc(x)\cot(x) + \frac{1}{2}\ln|\csc(x) - \cot(x)| + C$$

$\int \sin^2(x)dx = \frac{1}{2}x - \frac{1}{2}\sin(x)\cos(x) + C$, $\qquad \int \cos^2(x)dx = \frac{1}{2}x + \frac{1}{2}\sin(x)\cos(x) + C$

$\int x\ \sin(x)\ dx = \sin(x) - x\ \cos(x) + C$, $\qquad \int x\ \cos(x)\ dx = \cos(x) + x\ \sin(x) + C$

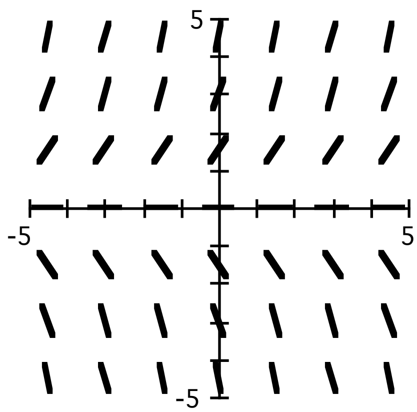
$\int e^{ax}\ \sin(bx)\ dx = \dfrac{e^{ax}\left(a\ \sin(bx) - b\ \cos(bx)\right)}{a^2 + b^2} + C$, $\int e^{ax}\ \cos(bx)\ dx = \dfrac{e^{ax}\left(a\ \cos(bx) + b\ \sin(bx)\right)}{a^2 + b^2} + C$

$\int \ln(x)\ dx = x\ln(x) - x + C$, $\qquad \int x\ e^{ax}dx = \frac{1}{a^2}(ax - 1)\ e^{ax} + C$

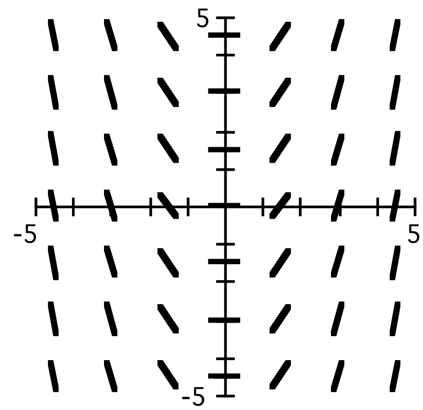
MAKE SURE YOU ANSWERED PROBLEM NUMBER 1.

2. Which of the following best represents the direction field of the differential equation $y' = y$ (7 pts)

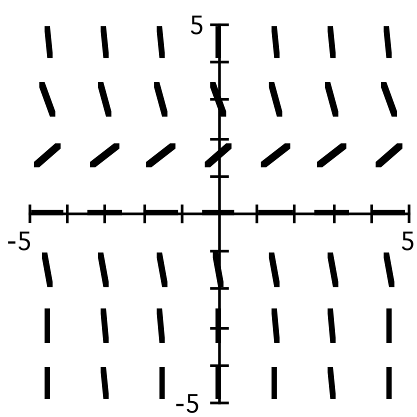
(**A**)



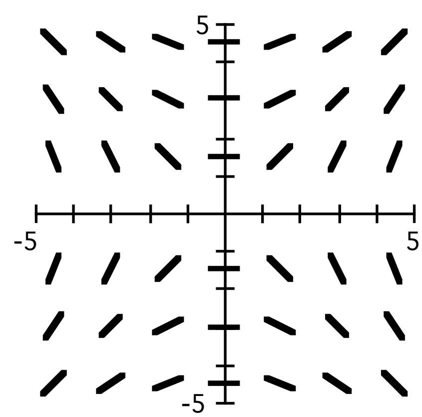
(**B**)



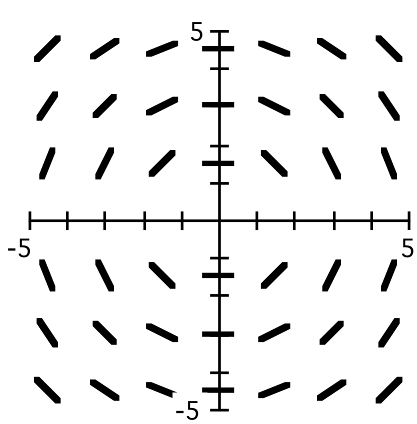
(**C**)



(**D**)



(**E**)



3. A tank contains 200 liters of fluid in which 30 g of salt is dissolved. Brine containing 1 g of salt per liter is then pumped into the tank at a rate of 4 liters per minute; the well-mixed solution is pumped out at the same rate. Which of the following initial value problems below would be used to find the number of grams of salt $A(t)$ in the tank at time t minutes,

(**A**) $\dfrac{dA}{dt} = 1 - 4A,\ \ A(0) = 30$

(**B**) $\dfrac{dA}{dt} = 4 - 0.02A,\ \ A(0) = 30$

(**C**) $\dfrac{dA}{dt} = 4 - A,\ \ A(0) = 30$

(**D**) $\dfrac{dA}{dt} = 50 - 4A,\ \ A(0) = 30$

(**E**) $\dfrac{dA}{dt} = 0.02 - 4A,\ \ A(0) = 30$

The differential equation $\dfrac{dy}{dt} = y^2(2t+3)$ is

4. (**A**) linear (**B**) Nonlinear

5. (**A**) separable (**B**) not separable

---

6. What is the largest interval on which a unique solution to the initial value problem $(t-6)y' + \ln(t)y = \sin(t)$, $y(3) = 2$ is guaranteed to exist.

(**A**) $(-\infty,\infty)$ (**B**) $(-\infty,6)$ (**C**) $(3,6)$ (**D**) $(0,6)$

(**E**) $(3,\infty)$

7. The differential equation $\dfrac{dy}{dt} = y(y-4)$ has

(**A**) unstable equilibrium solutions at $y=0$ and at $y=4$

(**B**) asymptotically stable solutions at $y=0$ and at $y=4$

(**C**) an unstable equilibrium solution at $y=0$ and an asymptotically stable solution at $y=4$

(**D**) an asymptotically stable solution at $y=0$ and an unstable equilibrium solution at $y=4$

(**E**) a semistable equilibrium solution at $y=0$ and an asymptotically stable solution at $y=4$

8. For what value of k is the following differential equation exact?
$\left(3x^2 - kx^2y + 2\right) + \left(6y^2 - 4x^3 + 3\right)\dfrac{dy}{dx} = 0$

(**A**) 0 (**B**) 3 (**C**) 6 (**D**) 12

(**E**) None of these

9. If $y'' - y' - y = 0$, $y(0) = 0$, $y'(0) = 0$ then $y(1)$ equals

(**A**) $\left(1+\sqrt{5}\right)e^{\frac{1+\sqrt{5}}{2}} + \left(1-\sqrt{5}\right)e^{\frac{1-\sqrt{5}}{2}}$ (**B**) $\sqrt{5}\ e^{\frac{1+\sqrt{5}}{2}} + \dfrac{\sqrt{5}}{2}\ e^{\frac{1-\sqrt{5}}{2}}$

(**C**) $e^{\frac{1+\sqrt{5}}{2}} - \sqrt{5}\ e^{\frac{1-\sqrt{5}}{2}}$ (**D**) $\sqrt{5}\ e^{\frac{1+\sqrt{5}}{2}} + \left(1-\sqrt{5}\right)\ e^{\frac{1-\sqrt{5}}{2}}$

(**E**) 0

10. Find the Wronskian, $W(y_1, y_2)$ for $y_1 = e^{-2t}$ and $y_2 = e^{3t}$.

(**A**) $5e^{6t}$ (**B**) $5e^{-6t}$ (**C**) $5e^{-5t}$ (**D**) $5e^{5t}$

(**E**) $5e^{t}$

11. Find the general solution to $y'' - 3y' + 2y = 0$

(**A**) $y = Ae^{-3t} + Be^{2t}$

(**B**) $y = e^{2t}\left[A\cos(3t) + B\sin(3t)\right]$

(**C**) $y = Ae^{2t} + Be^{t}$

(**D**) $y = Ae^{\frac{3t}{2}} + Bte^{\frac{3t}{2}}$

(**E**) $y = e^{3t}\left[A\cos(2t) + B\sin(2t)\right]$

12. Find the general solution to $y'' + 4y' + 5y = 0$

(**A**) $y = Ae^{4t} + Be^{5t}$

(**B**) $y = Ae^{5t} + Be^{-t}$

(**C**) $y = e^{-2t}\left[A\cos(t) + B\sin(t)\right]$

(**D**) $y = e^{-2t}\left[A\cos(2t) + B\sin(2t)\right]$

(**E**) $y = e^{5t}\left[A\cos(4t) + B\sin(4t)\right]$

13. Find the general solution to $t^2y'' + ty' - 6y = 0, \quad t > 0$

(**A**) $y = At^{-3} + Bt^{2}$

(**B**) $y = Ae^{-3t} + Be^{-2t}$

(**C**) $y = At^{\sqrt{6}} + Bt^{-\sqrt{6}}$

(**D**) $y = At^{\sqrt{6}} + Bt^{\sqrt{6}}\ln(t)$

(**E**) $y = t^{2}\left[A\cos(\sqrt{6}\ln(t)) + B\sin(\sqrt{6}\ln(t))\right]$

Place your final answer in the given rectangle. If your final answer is not in the rectangle, you will not receive full credit. An answer without proper justification will receive little if any credit.

14. Solve the initial value problem $\left(9x^2+y-1\right)+\left(x-4y\right)\dfrac{dy}{dx}=0$, $y(1)=2$

Place your final answer in the given rectangle. If your final answer is not in the rectangle, you will not receive full credit. An answer without proper justification will receive little if any credit.

15. Find the general solution to $ty' + 2y = t^3 + 2$