Potentially useful formulas: Math 256

$$y'' + p(t)y' + q(t)y = g(t) \qquad y = c_1 y_1 + c_2 y_2 + y_p$$

$$y_2 = u\ y_1 \qquad y_1 u'' + (2y_1' + p y_1)u' = 0$$

$$y_p = u_1 y_1 + u_2 y_2, \quad u_1' = \frac{W_1 g}{W}, \quad u_2' = \frac{W_2 g}{W}, \quad W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}, W_1 = \begin{vmatrix} 0 & y_2 \\ 1 & y_2' \end{vmatrix}, W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & 1 \end{vmatrix}$$

$$y''' + p(t)y'' + q(t)y' + r(t)y = g(t) \qquad y = c_1 y_1 + c_2 y_2 + c_3 y_3 + y_p$$

$$y_p = u_1 y_1 + u_2 y_2 + u_3 y_3 \qquad u_1' = \frac{W_1 g}{W}, \quad u_2' = \frac{W_2 g}{W}, \quad u_3' = \frac{W_3 g}{W},$$

$$W = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix}, \quad W_1 = \begin{vmatrix} 0 & y_2 & y_3 \\ 0 & y_2' & y_3' \\ 1 & y_2'' & y_3'' \end{vmatrix}, \quad W_2 = \begin{vmatrix} y_1 & 0 & y_3 \\ y_1' & 0 & y_3' \\ y_1'' & 1 & y_3'' \end{vmatrix}, \quad W_3 = \begin{vmatrix} y_1 & y_2 & 0 \\ y_1' & y_2' & 0 \\ y_1'' & y_2'' & 1 \end{vmatrix}$$

$$\int \sin(x)\ dx = -\cos(x) + C, \ \int \cos(x)\ dx = \sin(x) + C, \ \int \tan(x)dx = \ln|\sec(x)| + C, \ \int \cot(x)\ dx = -\ln|\csc(x)| + C$$

$$\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + C, \ \int \csc(x)dx = \ln|\csc(x) - \cot(x)| + C, \ \int \sec(x)\tan(x)dx = \sec(x) + C,$$

$$\int \csc(x)\cot(x)dx = -\csc(x) + C, \ \int \sec^2(x)dx = \tan(x) + C, \ \int \csc^2(x)dx = -\cot(x) + C$$

$$\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + C$$

$$\int \csc^3(x)dx = -\frac{1}{2}\csc(x)\cot(x) + \frac{1}{2}\ln|\csc(x) - \cot(x)| + C$$

$$\int \sin^2(x)dx = \frac{1}{2}x - \frac{1}{2}\sin(x)\cos(x) + C, \qquad \int \cos^2(x)dx = \frac{1}{2}x + \frac{1}{2}\sin(x)\cos(x) + C$$

$$\int x\ \sin(x)\ dx = \sin(x) - x\ \cos(x) + C, \qquad \int x\ \cos(x)\ dx = \cos(x) + x\ \sin(x) + C$$

$$\int e^{ax}\ \sin(bx)\ dx = \frac{e^{ax}\ \left(a\ \sin(bx) - b\ \cos(bx)\right)}{a^2\ + b^2} + C, \ \int e^{ax}\ \cos(bx)\ dx = \frac{e^{ax}\ \left(a\ \cos(bx) + b\ \sin(bx)\right)}{a^2\ + b^2} + C$$

$$\int \ln(x)\ dx = x\ln(x) - x + C, \qquad \int x\ e^{ax}\ dx = \frac{1}{a^2}(ax - 1)\ e^{ax} + C$$