Inverse Laplace Transforms
(via Table of Transform)

ex) $\mathcal{L}^{-1}\left(\frac{1}{s^2+4}\right) = \frac{1}{2}\mathcal{L}^{-1}\left(\frac{2}{s^2+4}\right)$

in table $\mathcal{L}(\sin at) = \frac{a}{s^2+a^2}$

$\mathcal{L}^{-1}\left(\frac{1}{s^2+4}\right) = \frac{1}{2}\sin 2t$ //

ex) $\mathcal{L}^{-1}\left(\frac{s-1}{s^2-s-6}\right)$ use Partial fractions

$$\frac{1}{s^2-s-6} = \frac{1}{(s-3)(s+2)} = \frac{A}{s+2} + \frac{B}{s-3}$$

$$1 = A(s-3) + B(s+2)$$

$A+B=0$ $\quad 1 = -3A+2B = -3A-2A$

$B=\frac{1}{5}$ $\quad A=-\frac{1}{5}$

$$\frac{s-1}{s^2-s-6} = -\frac{1}{5}\frac{s-1}{s+2} + \frac{1}{5}\frac{s-1}{s-3}$$

Perform long division:

$$= -\frac{1}{5}\left\{1 - \frac{3}{s+2}\right\} + \frac{1}{5}\left\{1 + \frac{2}{s-3}\right\}$$

$$= \frac{1}{5}\frac{3}{s+2} + \frac{1}{5}\frac{2}{s-3}$$

Table $\mathcal{L}(e^{at}) = \frac{1}{s-a}$

$$\mathcal{L}^{-1}\left(\frac{s-1}{s^2-s-6}\right) = \frac{3}{5}e^{-2t} + \frac{2}{5}e^{3t}$$

ex) $\mathcal{L}^{-1}\left(\frac{s}{s^2-4s+13}\right)$

$$\mathcal{L}^{-1}\left(\frac{b}{(s-a)^2+b^2}\right) = e^{at}\sin(bt)$$

$$\mathcal{L}^{-1}\left(\frac{s-a}{(s-a)^2+b^2}\right) = e^{at}\cos bt$$

Denominator: $s^2-4s+13 = s^2-4s+4-4+13$

$= (s-2)^2+9$ complete square

Numerator: $s = s-2+2$

$$\mathcal{L}^{-1}\left(\frac{s}{s^2-4s+13}\right) = \mathcal{L}^{-1}\left(\frac{s-2}{(s-2)^2+9}\right) + \mathcal{L}^{-1}\left(\frac{2}{(s-2)^2+9}\right)$$

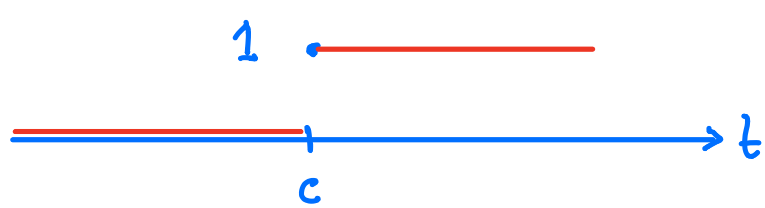
Using Table entries

$$= e^{2t}\cos 3t + \mathcal{L}^{-1}\left(\frac{2\cdot 3/3}{(s-2)^2+9}\right)$$

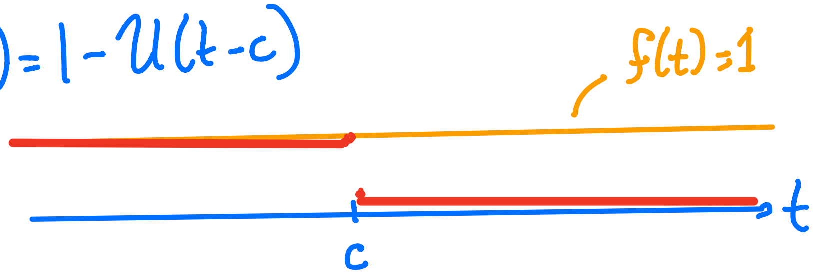
$$= e^{2t}\cos 3t + \frac{2}{3}e^{2t}\sin 3t$$

THE STEP FUNCTION (Heaviside Function)

$$\mathcal{U}(t-c) = \begin{cases} 0 & t < c \\ 1 & t \geq c \end{cases}$$

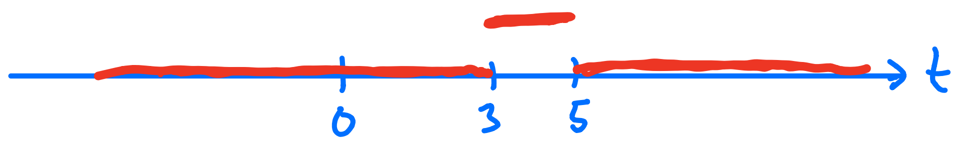


$$g(t) = 1 - \mathcal{U}(t-c)$$

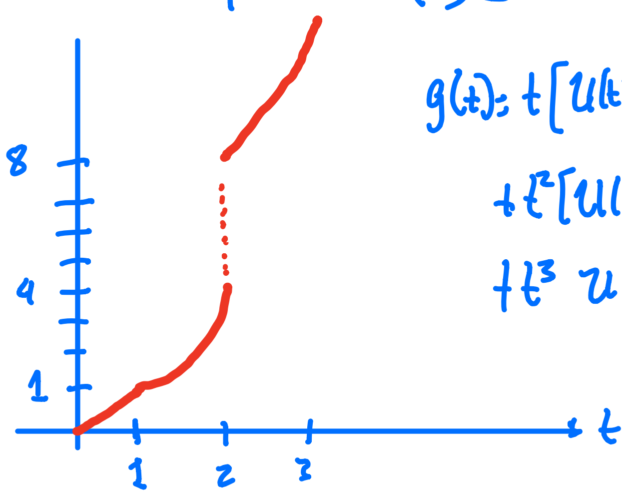


Represent $f(t) = \begin{cases} 0 & t<3 \\ 1 & 3 \le t < 5 \\ 0 & t \ge 5 \end{cases}$

$$f(t) = \mathcal{U}(t-3) - \mathcal{U}(t-5)$$



Ex) $g(t) = \begin{cases} t & 0 \le t < 1 \\ t^2 & 1 \le t < 2 \\ t^3 & t \ge 2 \end{cases}$



$$g(t) = t[\mathcal{U}(t) - \mathcal{U}(t-1)] + t^2[\mathcal{U}(t-1) - \mathcal{U}(t-2)] + t^3\,\mathcal{U}(t-2)$$