SECTION 1.3 CLASSIFICATION OF DIFFERENTIAL EQUATIONS

Recognizing the type of equation can lead you to a potential solution strategy. The classifiers will be explained mostly by example. They are:

* (scalar) equation, system of equation
* ODE, PDE
* ORDER
* LINEAR, NONLINEAR
* HOMOGENEOUS, NON HOMOGENEOUS

ORDINARY DE (ODE)

$\left\{ \begin{array}{l} \text{1 independent variable, 1 dep variables} \\ \text{"equation"} \\ \text{1 independent variable, several dependent variables} \\ \text{"system"} \end{array} \right.$

ex)

ODE $\quad \dfrac{d^2x}{dt^2} + \sin(t)x = f(t)$

here, $x = x(t)$.

SYSTEM OF ODE

$$\left\{ \begin{array}{l} \dfrac{dx}{dt} = f_1(x,y,z,t) \\ \dfrac{dy}{dt} = f_2(x,y,z,t) \\ \dfrac{dz}{dt} = f_3(x,y,z,t) \end{array} \right.$$

$x = x(t)$, $y = y(t)$, $z = z(t)$

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Partial Differential Equations (PDE

several independent variables, 1 dependent variable "equation"

several independent variables, several dependent variables (several equations) "system"

ex)

$$\frac{1}{c^2}\frac{\partial^2 u(x,t)}{\partial t^2} = \frac{\partial^2 u(x,t)}{\partial x^2}$$

here, $u = u(x,t)$

"The Wave Equation" (PDE)

ex)

$$\begin{cases} \dfrac{\partial u}{\partial t} = \nu_1 \left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} \right) + kuv \\[2ex] \dfrac{\partial v}{\partial t} = \nu_2 \left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} \right) - kuv \end{cases}$$

SYSTEM OF PDE'S

"Coupled Diffusion Equations"

$u = u(x,y,t)$, $v = v(x,y,t)$

ν_1, ν_2 and k are given parameters. //

We will focus entirely on ODE's

The ORDER: the order of an ODE describes the highest derivative present

ex) $$\frac{d^2y}{dt^2} + y = 0$$ $\left\{ \begin{array}{l} \text{second order} \\ \text{homogeneous} \\ \text{linear} \end{array} \right.$

(Homogeneity & linearity will be explained later)

ex) $$y\frac{d^3y}{dt^3} - \frac{dy}{dt} = 1$$ 3rd order ODE
non-homogeneous
nonlinear

ex)
$$\boxed{\begin{array}{c} a_n \frac{d^{(n)}y}{dx^{(n)}} + a_{n-1}\frac{d^{(n-1)}}{dx^{(n-1)}}y + \dots + a_2\frac{d^2y}{dx^2} \\ + a_1\frac{dy}{dx} + a_0 y = 0 \qquad (\dagger) \\ a_n \neq 0, \; a_i \;\; i=0,1,2\dots,n-1 \text{ not all zero.} \end{array}}$$

n^{th} order linear homogeneous equation

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Let

$$\mathcal{L} \equiv a_n(x)\frac{d^{(n)}}{dx^{(n)}} + a_{n-1}(x)\frac{d^{(n-1)}}{dx^{(n-1)}} + \dots + a_1\frac{d}{dx} + a_0$$

$$\mathcal{L}y = 0 \quad \text{is} \quad (7)$$

where $a_i = a_i(x)$ $i = 0, 1, 2, \dots, n$

a_i are not all zero

a_n nonzero

HOMOGENEOUS VS NON HOMOGENEOUS EQUATION

$\mathcal{L}y = 0$ Homog. linear n^{th} order ODE

$y(x)$ the right hand side is zero

$$\mathcal{L}y = f(x)$$

Non-Homogeneous counterpart.

not a function of y!