Section 1.1

What is an ordinary differential equation?

equations that feature derivatives of variables are differential equations.

They are ordinary, as opposed to partial, because no partials are featured.

ODE (ordinary), PDE (partial) differential equation

ex) S'pse $y = y(x)$. An equation

$$g\left(x, y(x), \frac{dy}{dx}(x), \frac{d^2y}{dx^2}(x), \dots, \frac{d^ny}{dx^n}\right) = 0 \quad (\dagger)$$

is an ODE. One or several derivatives of y appear in g.

e.g. $3y'' + 2y' = 0$, $y = y(x)$

and $(\ )' = \frac{d}{dx}$, $(\ )'' = \frac{d^2}{dx^2}$

Here x is the "independent variable" and y is the dependent variable.

ex) $\frac{dy}{dt} = t^2$. Using (‡)

$y = y(t)$, t is independent variable.

$$g\left(t, y, \frac{dy}{dt}\right) = \frac{dy}{dt} - t^2 = 0$$

Can also be that $t = t(y)$ and

$$h\left(y, t, \frac{dt}{dy}\right) = 0$$

is $\frac{dt}{dy} = \frac{1}{t^2}$, so

$$h = \frac{dt}{dy} - \frac{1}{t^2} = 0$$ //

eg) $$\frac{dv}{dt} = g - \beta v, \quad \left.\begin{matrix} g \\ \beta \end{matrix}\right\} \text{constants}$$

if $v = v(t)$ speed

$$v = \frac{[L]}{[T]}$$ Length / Time

$$\frac{dv}{dt} = \frac{[L\,T^{-1}]}{[T]} = \frac{[L]}{[T^2]}$$

the acceleration.

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Focus on 1st Order ODE:

$$\frac{dy}{dt} = \underbrace{f(t, y(t))}_{\text{rate function}}$$

$$y = y(t)$$

$$g\left(t, y, \frac{dy}{dt}\right) = \frac{dy}{dt} - f(t, y) = 0$$

At each $(t, y(t))$, we are given the rate of change of y wrt t.

ex) $$\frac{dy}{dt} = t$$

Integrate both sides wrt t

$$y(t) + c_1 = \frac{1}{2}t^2 + c_2$$

constant

$$y(t) = \frac{1}{2}t^2 + C$$

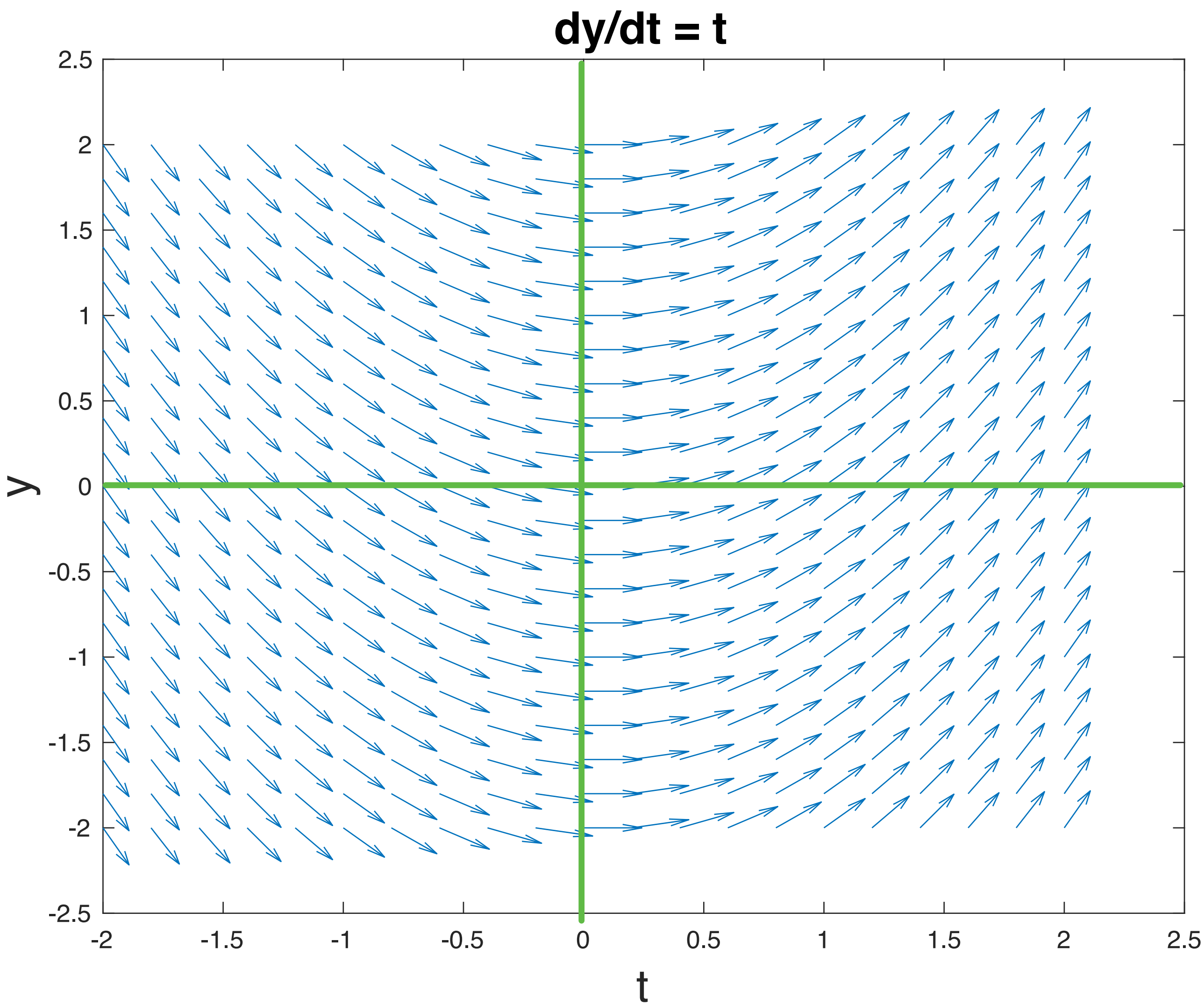
$$C = C_2 - C_1$$

Can approximate graphically

:

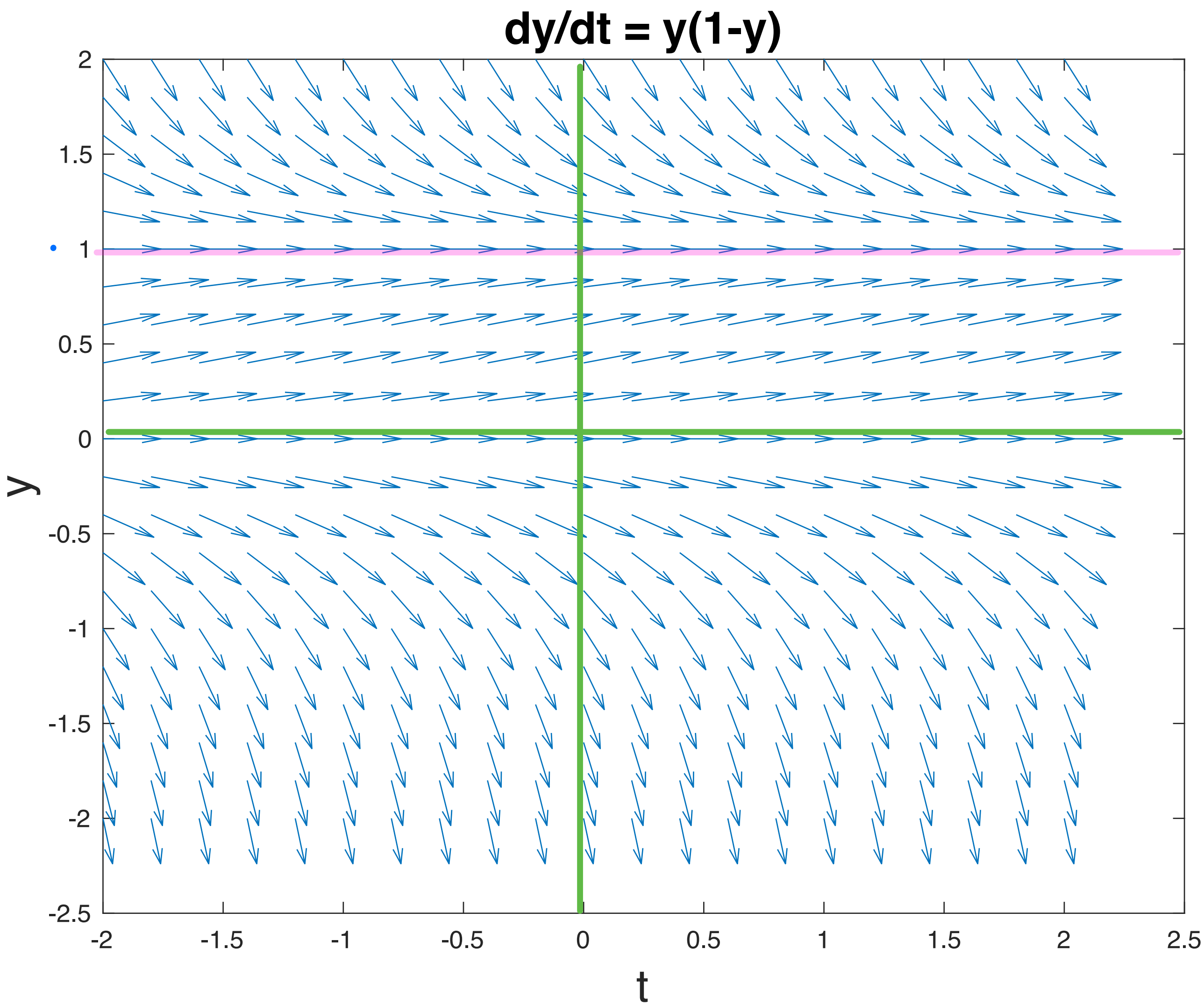
$$\frac{dy}{dt} = t = f(t, y)$$

lx)



$y = \frac{1}{2}t^2 + c$

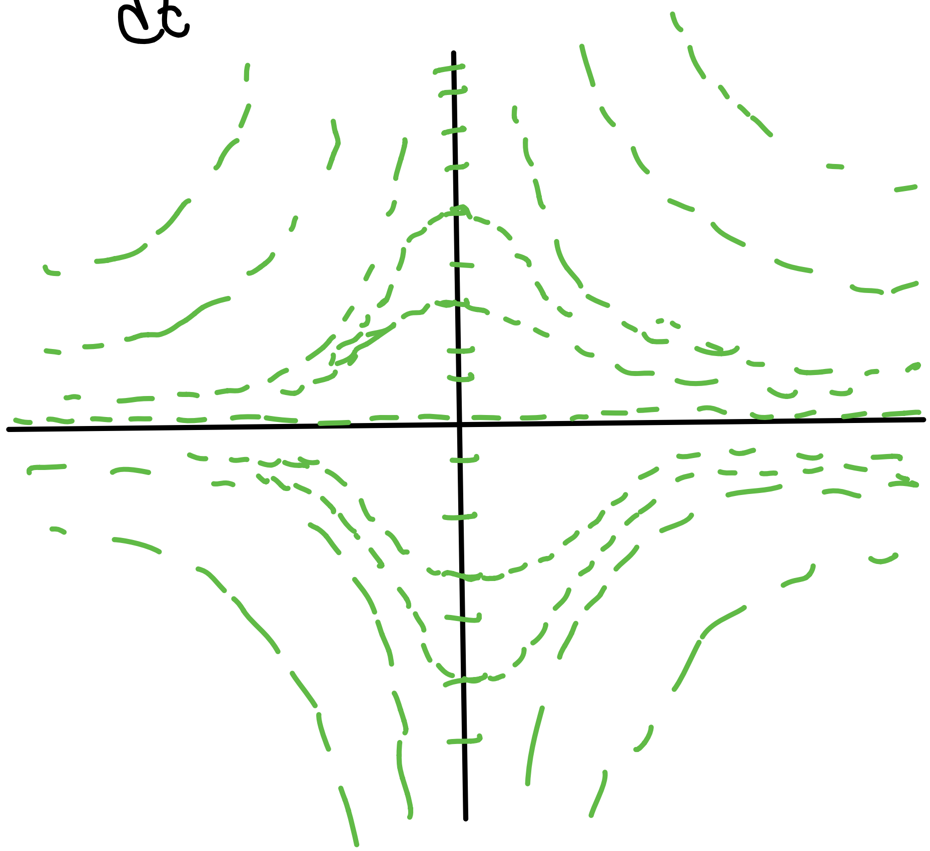
ex)



$y=1$ is an "equilibrium solution"

$y=0$ is also an equilibrium solution

ex) $\frac{dy}{dt} = -ty$



Mathematical Models can be ODE's. For example ex) Population of bacteria

① Observation: collect facts from experiment

Question: What is the rate of growth of the bacteria?

② Identify dependent & independent variable(s) associated with question.

let $p(t)$ be # of bacteria

$p(t) \geq 0$ (assuming the population is large so that it can be measured continuously. Note: the bacteria should take discrete values, so $p(t)$ continuous is an approximation.

let t be time (sec)

p is the dependent variable

t is the independent variable.

Data

t [sec]	P (#)	$\frac{dp}{dt}$ (#/sec)	$\frac{1}{p}\frac{dp}{dt}$ (1/sec)
1	1	1	$\frac{1}{2}$
2	2	2	$\frac{1}{2}$
3	4	4	$\frac{1}{2}$
4	8	8	$\vdots$
5	16	16	
6	32	$\vdots$	
$\vdots$	$\vdots$		

Codify the structure of the results:

$$\frac{1}{p}\frac{dp}{dt} = k = \frac{1}{2} \quad \text{(a constant)}$$

units $\frac{1}{[\#]}\frac{[\#]}{[sec]} = \frac{1}{[sec]}$

Model $\frac{dp}{dt} = kp \qquad t \geq 1$

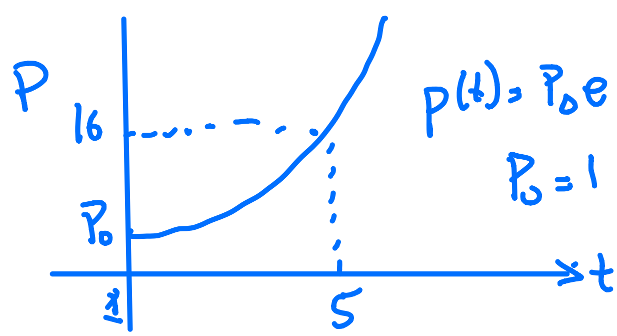
($) $\dfrac{dp}{dt} = \dfrac{1}{2} P$ since $k = \dfrac{1}{2}$

$t > 1$

$p(t) = P_0 e^{\frac{1}{2}(t-1)}$ is the solution to ($)

P_0 is the initial population

$p(t=0) = P_0$ the initial population



$p(t) = P_0 e^{\frac{1}{2}(t-1)}$

$P_0 = 1$

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P_0 is pinned down with another equation I.C. initial condition