$\vec{A} = (3x^2 + 2y^2)\hat{\imath} + (4xy + 6y^2)\hat{\jmath}$

Is it conservative?

Is curl $\vec{A} = 0$?

Curl $\vec{A} = \nabla \times A$

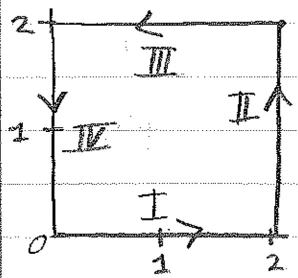
$$\begin{matrix} \hat{\imath} & \hat{\jmath} & \hat{k} & \hat{\imath} & \hat{\jmath} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z & \partial/\partial x & \partial/\partial y \\ 3x^2+2y^2 & 4xy+6y^2 & 0 & 3x^2+2y^2 & 4xy+6y^2 \end{matrix}$$

$-4y\hat{k} - 0\hat{\imath} - 0\hat{\jmath} \qquad 0\hat{\imath} + 0\hat{\jmath} + 4y\hat{k}$

Curl $\vec{A} = 0$, So $\vec{A}$ is conservative, This means that for a closed path,

$\oint_c \vec{A} \cdot dr = 0$

Path 1: $(0,0) \to (2,2) \to (0,2) \to (0,0)$



$\vec{A} = (3x^2 + 2y^2)\hat{\imath} + (4xy + 6y^2)\hat{\jmath}$

$d\vec{r} = dx\,\hat{\imath} + dy\,\hat{\jmath}$

$$\oint_c \vec{A}\cdot d\vec{r} = \int_I \vec{A}\cdot d\vec{r} + \int_{II} \vec{A}\cdot d\vec{r} + \int_{III} \vec{A}\cdot d\vec{r} + \int_{IV} \vec{A}\cdot d\vec{r}$$

I) $0 \le x \le 2$, $y = 0$

$$\int_I A\cdot dr = \int_0^2 3x^2 + \cancel{2y^2}^{\,0}\, dx + \cancel{\int_0^0 4xy + 6y^2\, dy}^{\,0}$$

$$= x^3\Big]_0^2 = 8$$

II) $x=2,\ 0 \le y \le 2$

$$\int_{II} A\cdot dr = \int_2^2 \cancel{3x^2+2y^2}\,dx \to 0 + \int_0^2 4xy+6y^2\,dy$$

$$= 2xy^2+2y^3\Big]_0^2 = 16+16=32$$

III) $y=2$, x goes from 2 to 0

$$\int_{III} A\cdot dr = \int_2^0 3x^2+2y^2\,dx + \int_2^2 \cancel{4xy+6y^2}\,dy \to 0$$

$$= x^3+2y^2x\Big]_2^0 = -8-16=-24$$

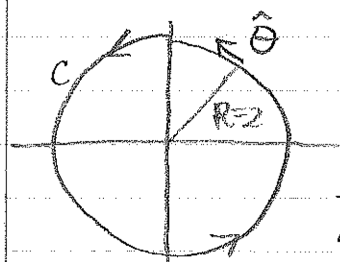
IV) $x=0$, y goes from 2 to 0

$$\int_{IV} A\cdot dr = \int_0^0 \cancel{3x^2+2y^2}\,dx \to 0 + \int_2^0 \cancel{4xy}^{\,0}+6y^2\,dy$$

$$= 2y^3\Big|_2^0 = -16$$

$$\int_C \vec{A}\cdot d\vec{r} = 8+32-24-16=0$$

Path 2: Circle w/ $R=2$, counterclockwise from 0 to 2π



Let $x=2\cos\theta$
$y=2\sin\theta$

$$\vec{A}=(16\cos^2\theta+4\sin^2\theta)\hat{\imath}+(16\cos\theta\sin\theta+24\sin^2\theta)\hat{\jmath}$$

$$\hat{\theta}=-\sin\theta\,\hat{\imath}+\cos\theta\,\hat{\jmath}$$

$$dr=\hat{\theta}\,d\theta$$

$$\oint_C A\cdot dr = \int_0^{2\pi} A\cdot\hat{\theta}\,d\theta$$

$A \cdot \hat{\theta} = -6\cos^2\theta\sin\theta - 4\sin^3\theta + 16\cos^2\theta\sin\theta + 24\sin^2\theta\cos\theta$

$A \cdot \hat{\theta} = 10\cos^2\theta\sin\theta - 4\sin^3\theta + 24\sin^2\theta\cos\theta$

$$\int_0^{2\pi} A \cdot \hat{\theta}\, d\theta = \int_0^{2\pi} (10\cos^2\theta\sin\theta - 4\sin^3\theta + 24\sin^2\theta\cos\theta)\, d\theta$$

$$= -\frac{10}{3}\cos^3\theta\Big|_0^{2\pi} \to 0 - 4\left[\frac{-3\cos\theta}{4} + \frac{\cos(3\theta)}{12}\right]_0^{2\pi} \to 0$$

$$+ 8\sin^3\theta\Big|_0^{2\pi} \to 0$$

$$\oint A \cdot dr = 0$$