MATH 251 FINAL EXAM 200 pts

NAME:

Instructions: No collaboration. You must include all the steps in your derivations/answers. Reduce answers as much as possible, but use *exact arithmetic*. Write neatly.

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1. (50) Find the derivative of

$$f(x) = \sin[\tan^{-1}(\ln x)].$$

2. (50) Consider the following function:

$$f(x) = \frac{x^2}{x^2 - 4}.$$

Use plots as well as analysis to:

(a) identify asymptotes, if any.

(b) Find the domain and range of the function.

(c) Identify symmetry, if any.

(d) Find critical points, if any.

(e) Find inflection points, if any.

(f) identify regions of up/down concavity, if any.

(g) Find local maxima, minima.

(h) Find global maxima, minima, if any.

3. (50) Find the $\lim_{x\to 0}[x^p \ln(x)]$, for p positive.

4. (50) You want to build a barrier using fencing material. There is 400 m of fencing available. You decide to build the pen against an already existing straight tall wall, thus no fencing is needed on the side with the wall. If you want to maximize the area of the pen and the pen is rectangular (on 3 sides anyway), what dimensions should the pen have?

5. (optional, up to 10 extra credit points) Let $f(x)$ and $g(x)$ be 2 continuous functions. Suppose as well that $f''(x_0) > 0$ and $g''(x_0) > 0$. Is the second derivative of the product necessarily positive? that is,

$$\frac{d^2[fg](x_0)}{dx^2}$$

necessarily positive? Prove or give a counterexample.