(0) Each problem will increase your midterm grade by at most 10/100. Messy work will affect your grade. Problem (1) is compulsory.

(1) Redo the midterm. You will need to get every problem right and all answers must be well-reasoned and complete.

(2) Calculate the following limits by expressing each one as the derivative of some function:

a) $\lim_{x \to 1} \dfrac{x^8 - x^7 + 3x^2 - 3}{x-1}$

b) $\lim_{x \to 2} \dfrac{1/x^3 - 1/2^3}{x-2}$

c) $\lim_{x \to -1} \dfrac{x^2 + x}{(x+2)(x+1)}$

(2) Differentiate $(1+(1+(1+x^2)^8)^8)^8$
(you need to show all steps, to get credit).

③ A population is said to grow in time

$$p(t) = \frac{K}{1+Ae^{-\alpha t}} \quad , t>0$$

where K is a positive constant, $\alpha>0$

$$A = \frac{K-P_0}{P_0} \quad , K>P_0$$

P_0 is the population at $t=0$, $P_0>0$.

a) Plot a typical $p(t)$, as a function of t.

Use $\alpha = 10^{-2}$ $K=10^5$ $P_0 = 10^3$

a.1 $t=[0,200]$
describe $p(t)$

a.2 $t=[0,600]$
describe $p(t)$

a.3 $t=[0,2000]$
describe $p(t)$

By describing we mean interpret what this model is saying about the population.

(b) $\lim_{t \to \infty} p(t)$

(c) We will compute the sensitivity of the population to changes in time t:

If $p(t + \delta t)$ is a perturbation of the population $p(t)$ at time t, then

$\frac{\delta p}{p}$ is the relative sensitivity of the population (relative to p) at time t.

$$s = \frac{\delta p}{p} = \frac{\delta t}{p} \frac{dp}{dt}$$

A small perturbation of time δt, at time t, will lead to a relative population sensitivity given by the above formula.

C1. Show that $\frac{\delta p}{\delta t} \approx \frac{dp}{dt}$, for some t.

You use a figure. For what t is the approximation better?

C2. Compute $\frac{dp}{dt}$ analytically.

C3. Plot $s(t) = \frac{t}{P} \frac{dp}{dt}$ as a function of t.

Interpret this plot as a sensitivity of the population at different times

C4. Assume that the population you are tracking is a crop pest. Can you use the estimate from C3 to advise a farmer regarding when would be a best <u>period</u> of time to treat his/her field?

Take into account the following:

- $t > 0$

- It can be assumed that the treatment is effective in reducing the population, but not 100% effective.

- The population is spread uniformly across the field

- The treatment can be spread uniformly on the field.

- The treatment affects the pest by disrupting reproduction.

- The treatment is only effective for a short time after its application.