

Carbon-based (organic) materials.

PH 673

Nanoscience and nanotechnology

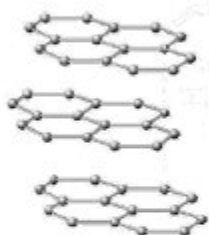
October 29, 2025

Carbon

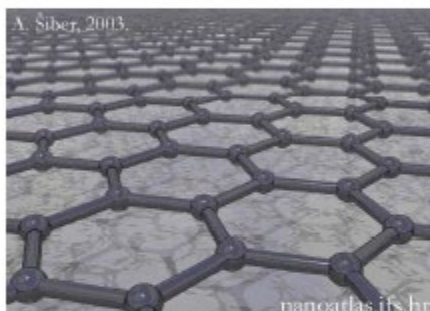
3d



Graphite



2d



Graphene

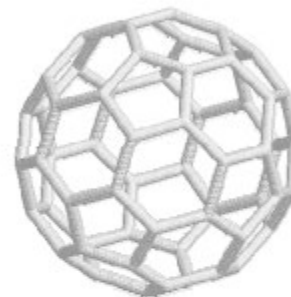
1d



Nanotube

"Multi-wall" 1991
"Single-wall" 1993

0d

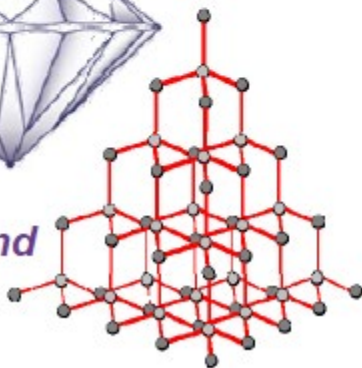


Fullerene

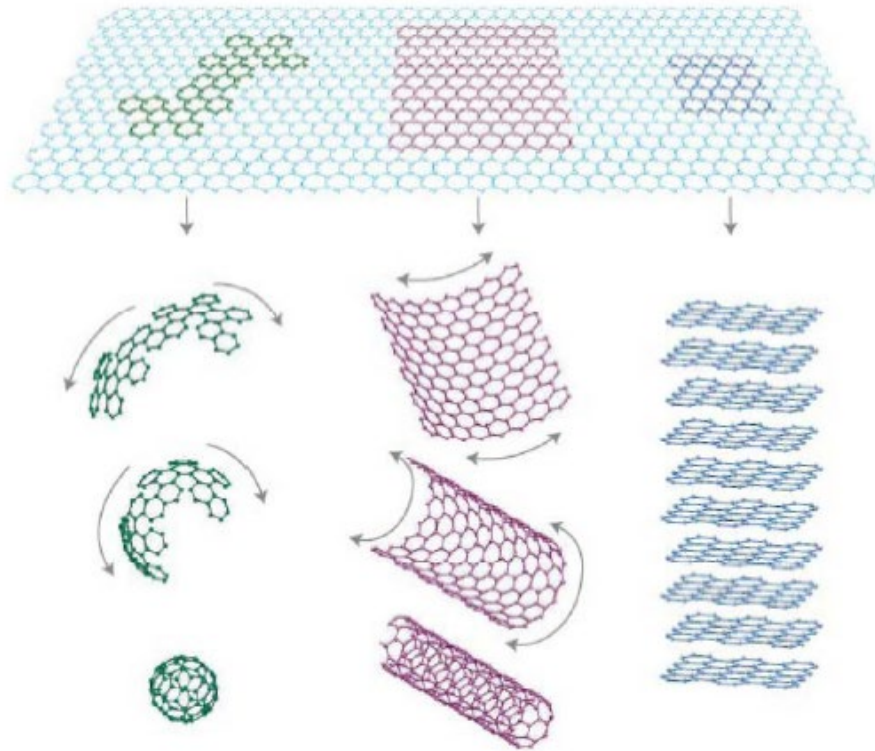
1985



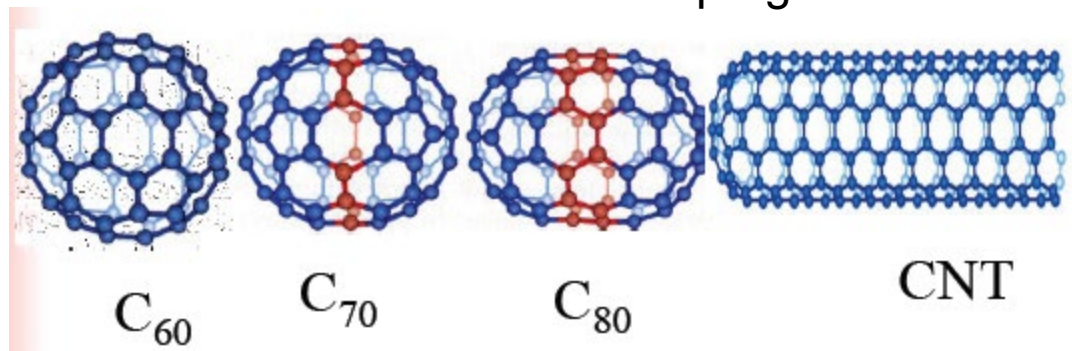
Diamond



graphene, fullerenes, nanotubes and graphite

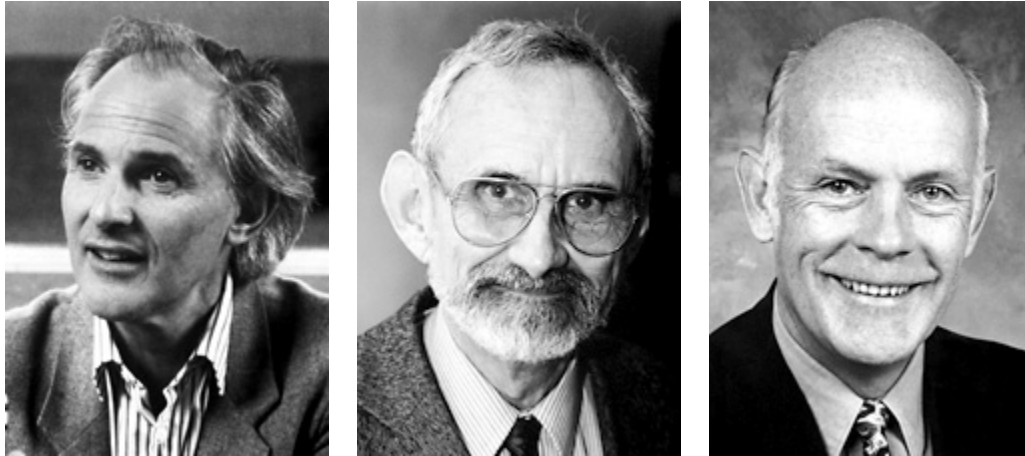


Or... fullerene progression



Nobel prize in Chemistry 1996

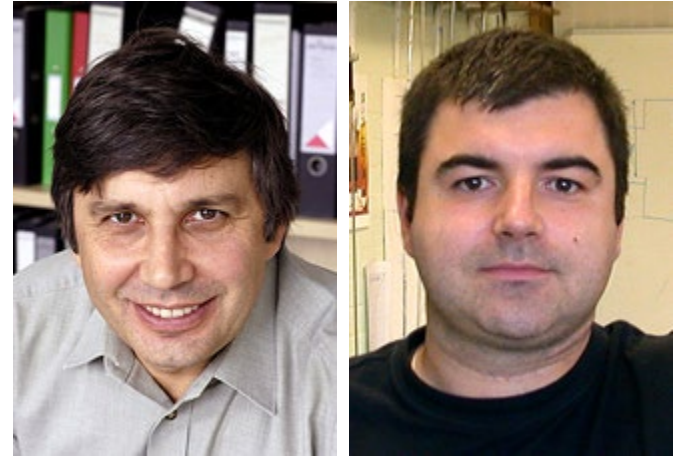
Fullerenes



Robert F. Curl Jr. (Rice U, USA), Sir Harold W. Kroto (U of Sussex, UK) and Richard E. Smalley (Rice U, USA) *"for their discovery of fullerenes"*.

Nobel prize in Physics 2010

Graphene

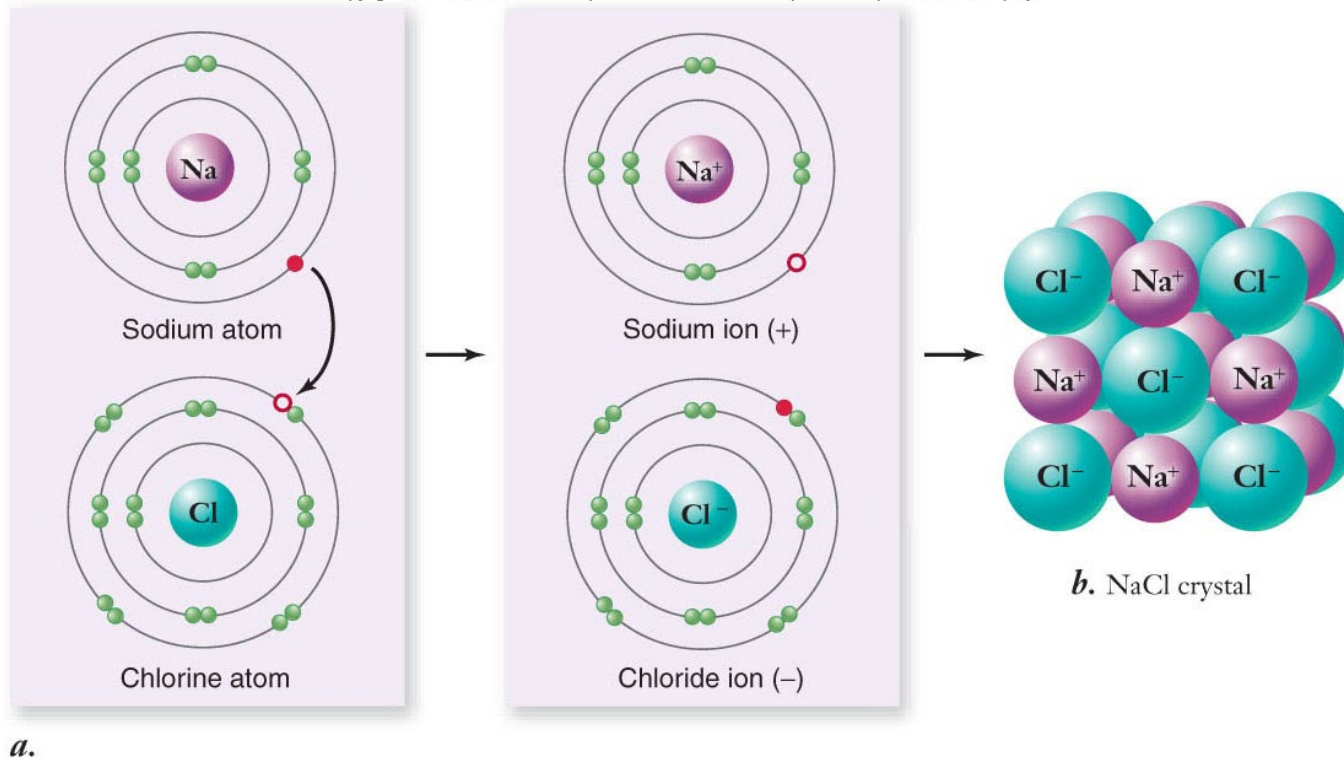


Andre Geim and Konstantin Novoselov (U of Manchester, UK) *"for groundbreaking experiments regarding the two-dimensional material graphene"*

Chemical Bonds

Ionic bonds are formed by the attraction of oppositely charged ions.

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Periodic Table of the Elements

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Key

- 1 — atomic number
- H — chemical symbol

1 H																	2 He
3 Li	4 Be															9 F	10 Ne
11 Na	12 Mg															17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	57 La	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	89 Ac	104 Rf	105 Ob	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Uuu	112 Uub	113 Uut	114 Uuq	115 Uup	116 Uuh	117	118
(Lanthanide series)		58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu		
(Actinide series)		90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr		

a.

Chemical Bonds

Covalent bonds form when atoms share 2 or more valence electrons.

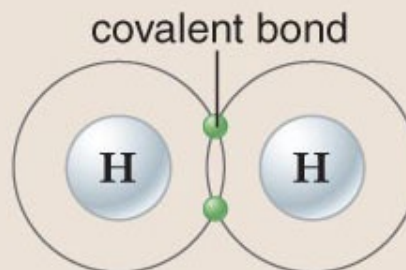
Covalent bond strength depends on the number of electron pairs shared by the atoms.

single bond < double bond < triple bond

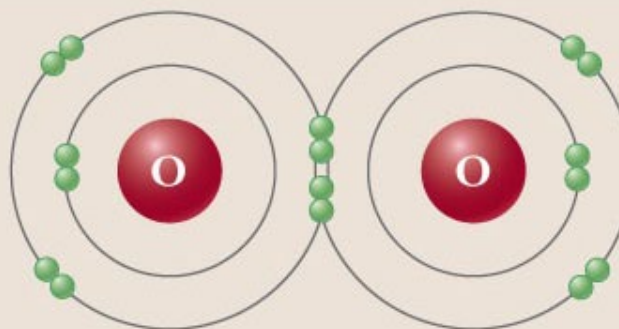
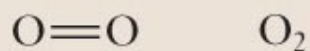
Chemical Bonds

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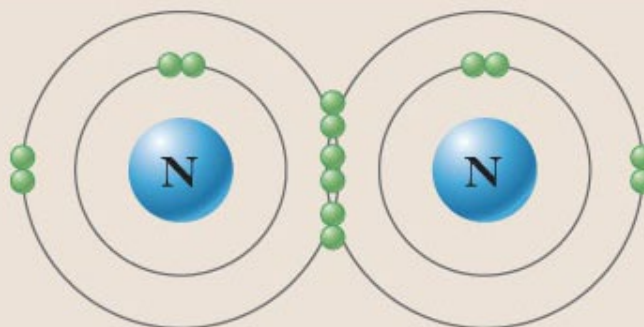
Single covalent bond
hydrogen gas



Double covalent bond
oxygen gas



Triple covalent bond
nitrogen gas



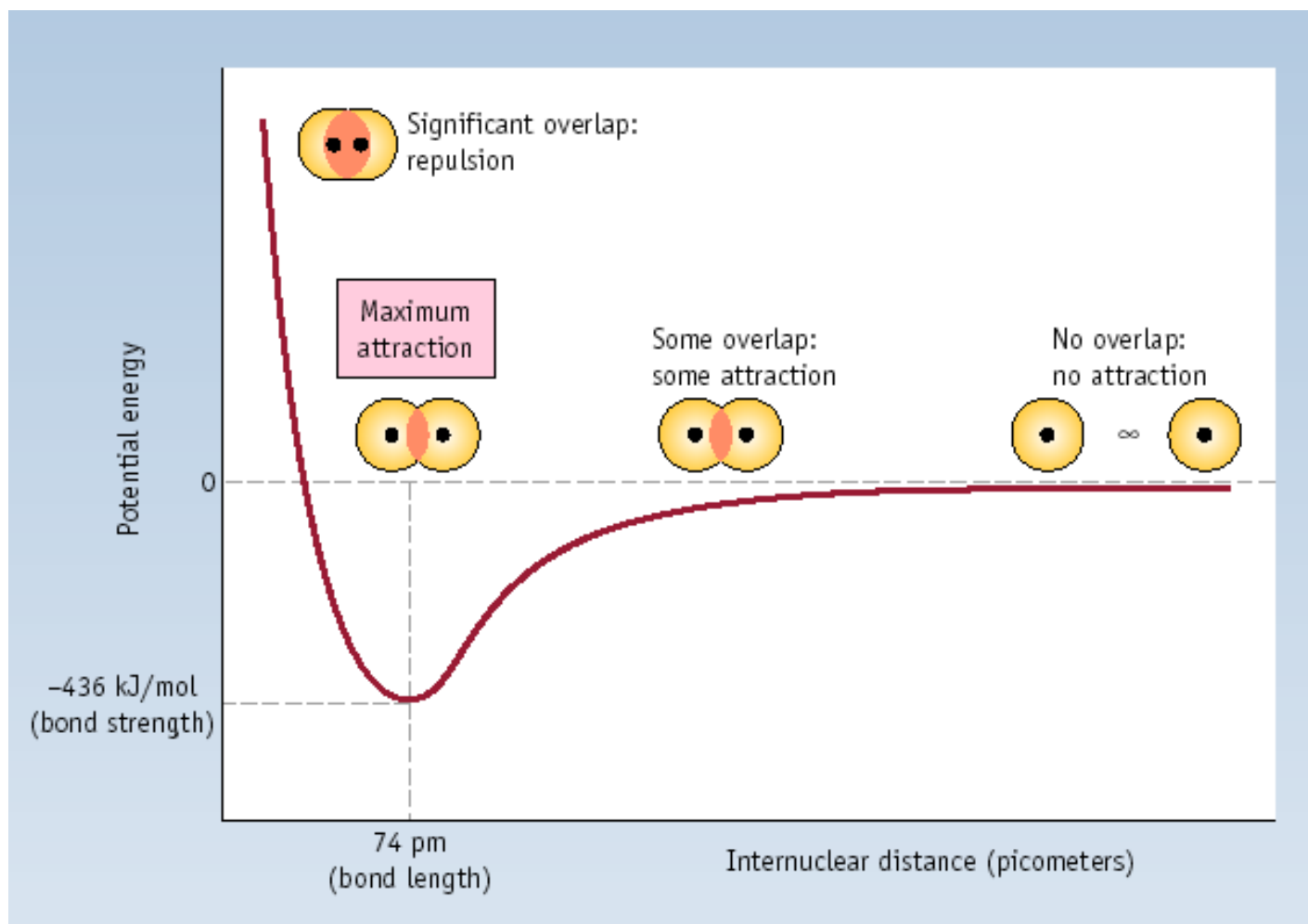
Chemical Bonds

Electronegativity is an atom's affinity for electrons.

Differences in electronegativity dictate how electrons are distributed in covalent bonds.

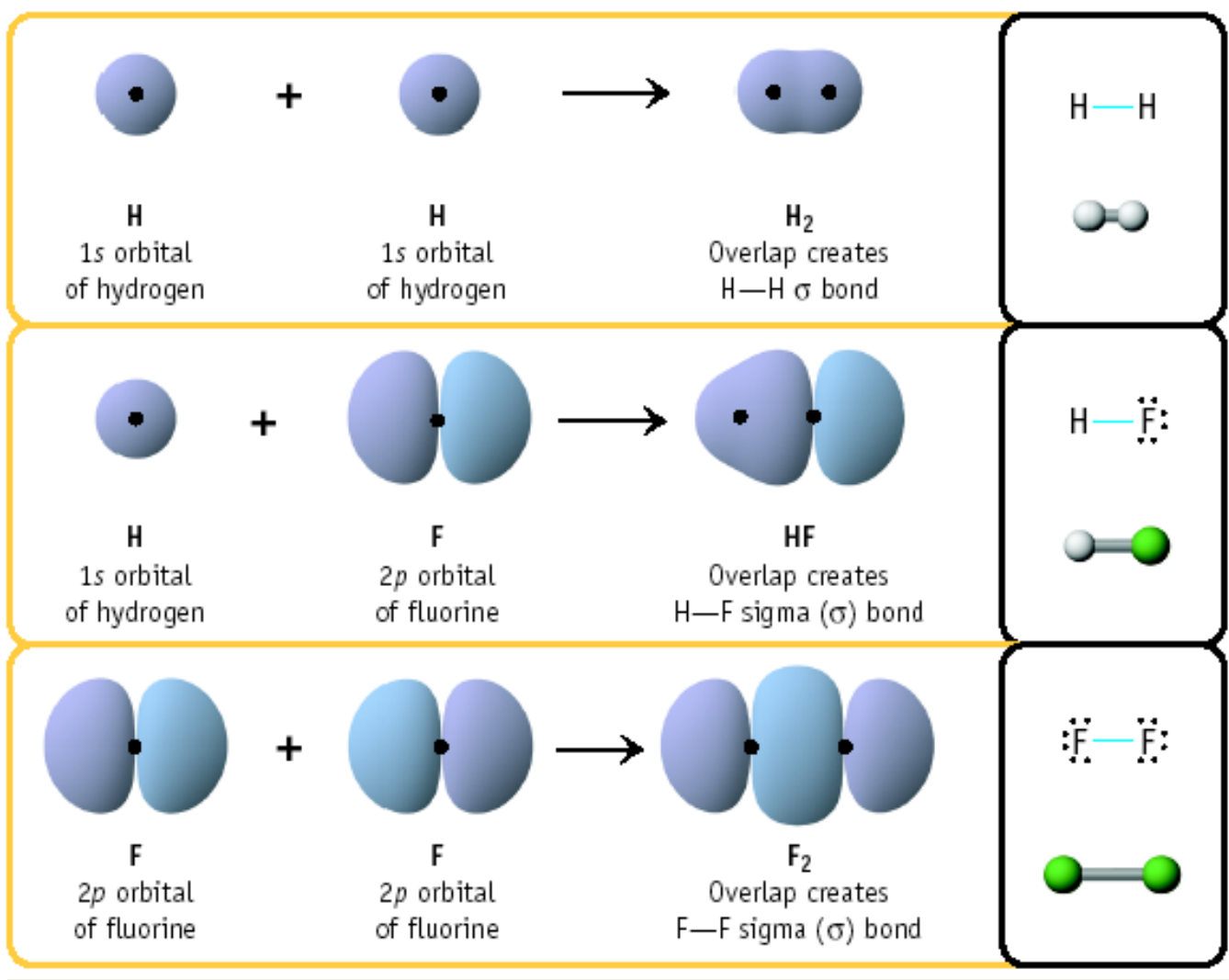
- **nonpolar covalent bonds** = equal sharing of electrons
- **polar covalent bonds** = unequal sharing of electrons

Energy Diagram of Sigma Bond Formation by Orbital Overlap

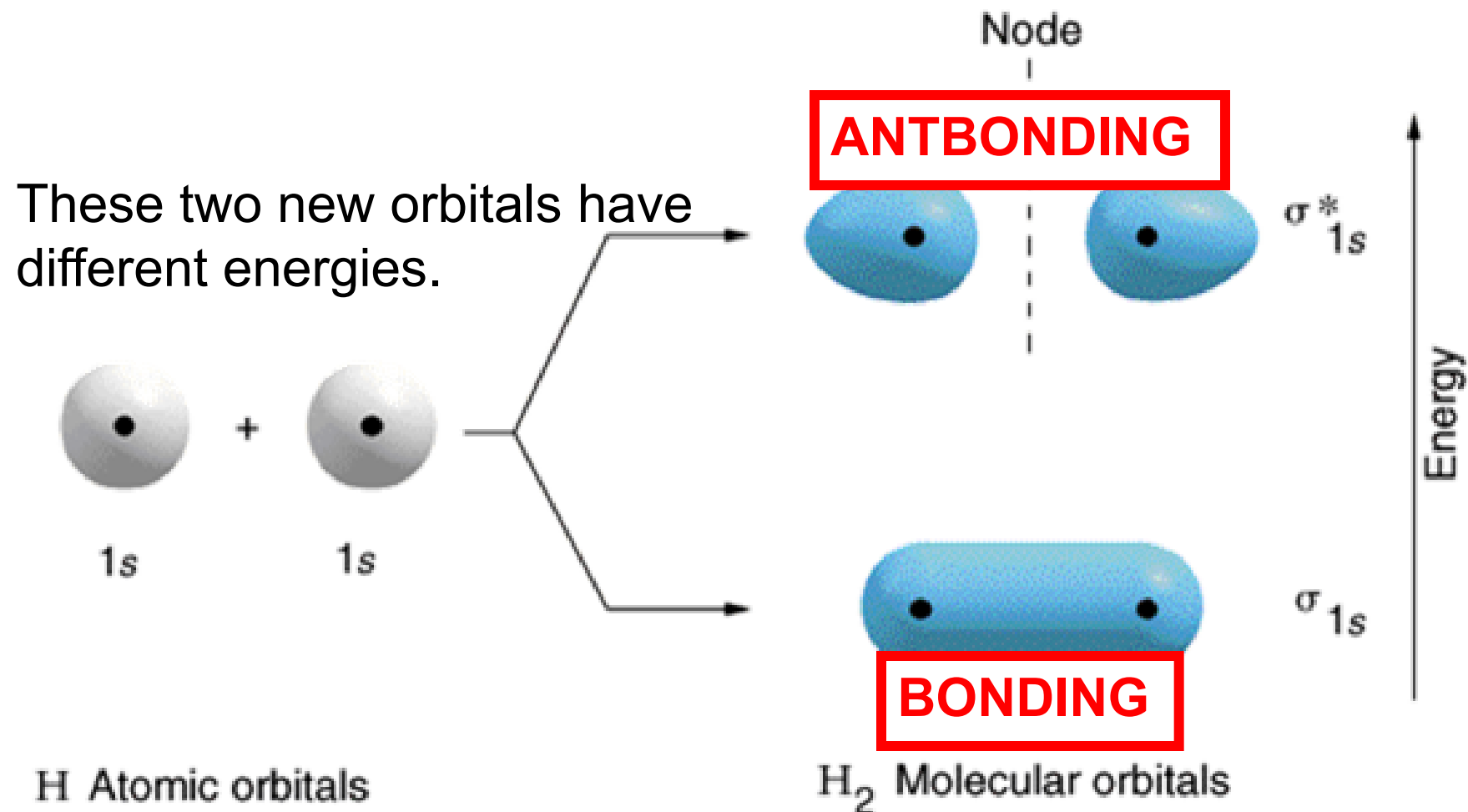


Examples of Sigma Bond Formation

Sigma bond (σ) $\rightarrow$ A bond where the line of electron density is concentrated symmetrically along the line connecting the two atoms.



Molecular Orbital (MO) Theory

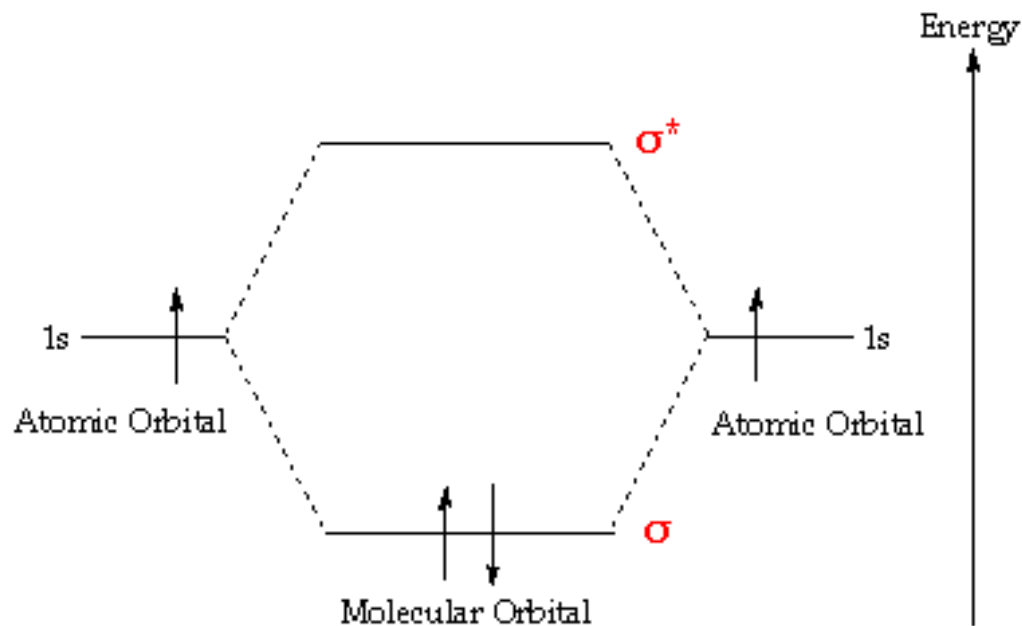


The one that is lower in energy is called the **bonding orbital**,

The one higher in energy is called an **antibonding orbital**.

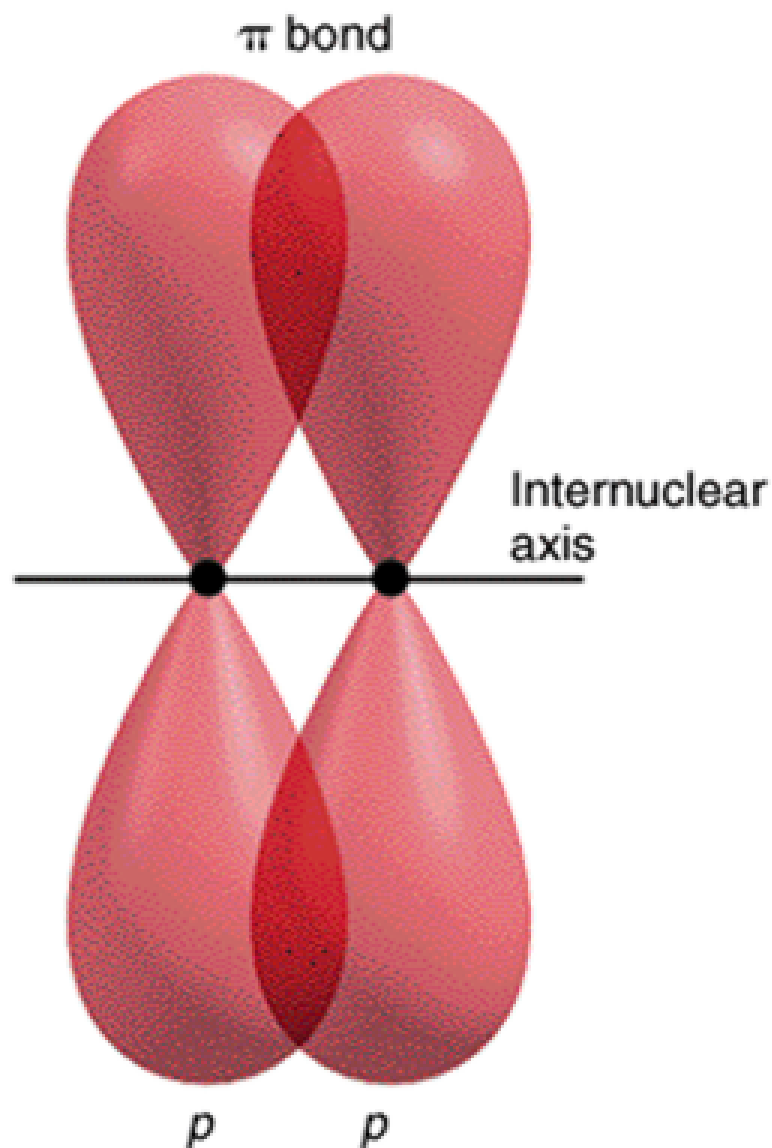
Atomic and Molecular Orbitals

- In atoms, electrons occupy **atomic orbitals**, but in molecules they occupy similar **molecular orbitals** which surround the molecule.
- The two 1s atomic orbitals combine to form two molecular orbitals, one bonding (σ) and one antibonding (σ^*).



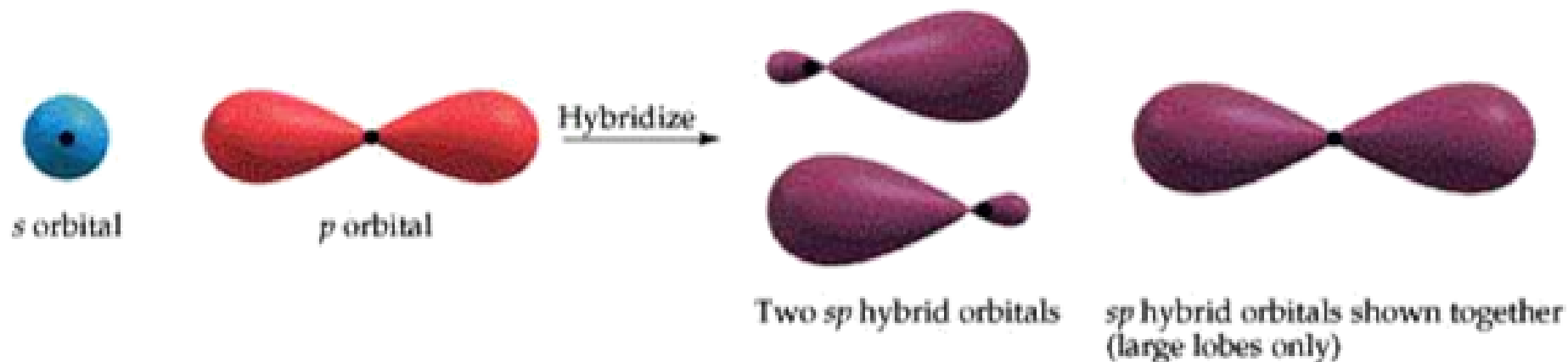
- This is an illustration of molecular orbital diagram of H_2 .
- Notice that one electron from each atom is being “shared” to form a covalent bond. This is an example of orbital mixing.

Pi bond (π) → A bond where the overlapping regions exist above and below the internuclear axis (with a nodal plane along the internuclear axis).



Formation of sp hybrid orbitals

The combination of an s orbital and a p orbital produces 2 new orbitals called sp orbitals.

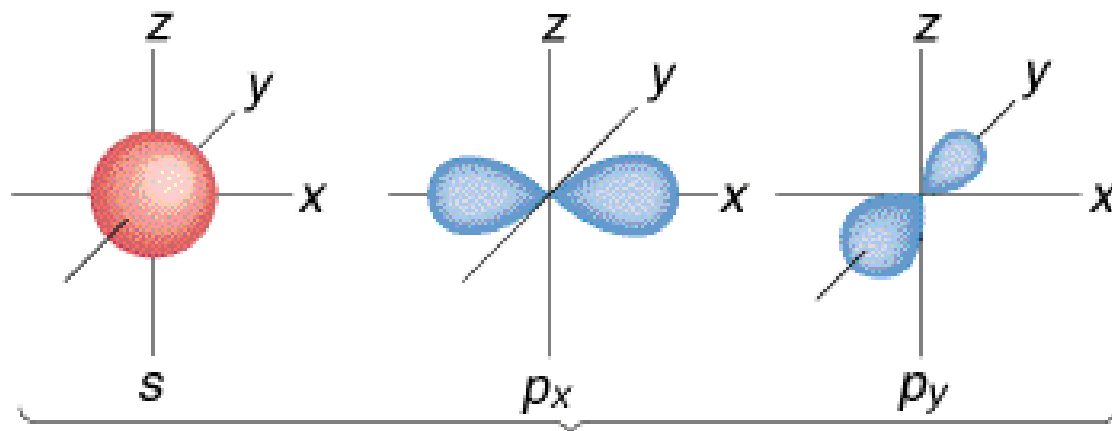


These new orbitals are called **hybrid orbitals**

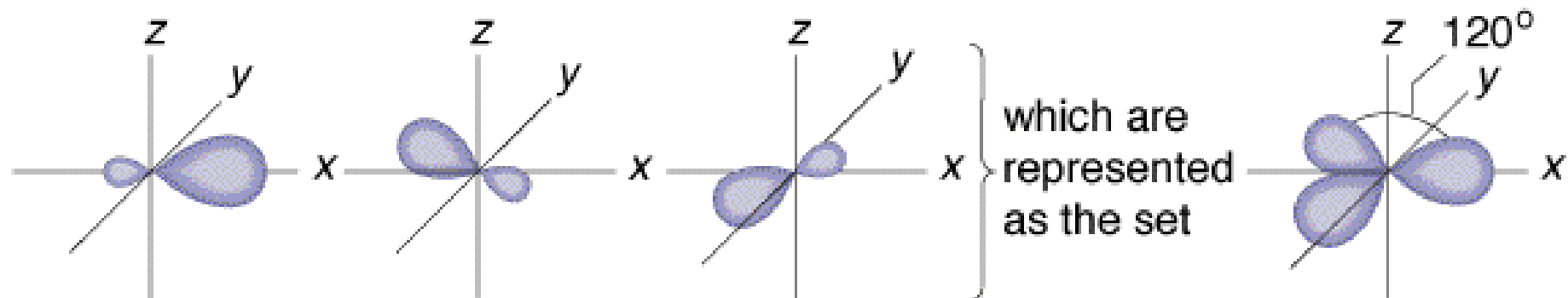
The process is called **hybridization**

What this means is that both s and p orbitals are involved in bonding to the connecting atoms

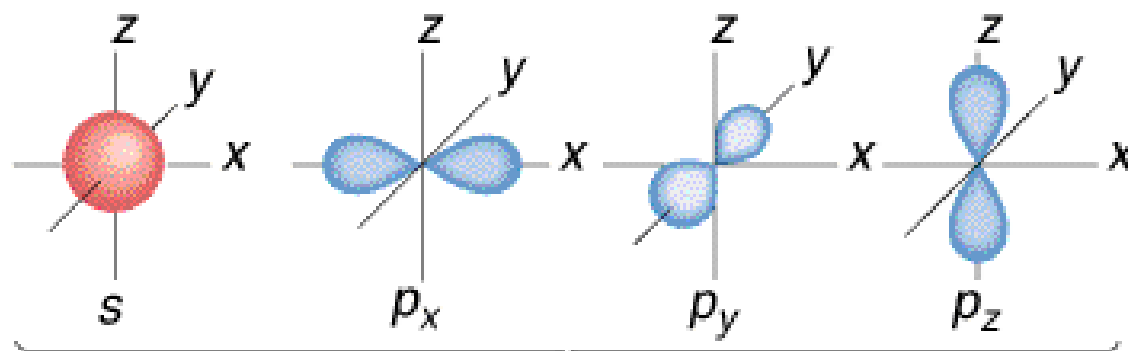
Formation of sp^2 hybrid orbitals



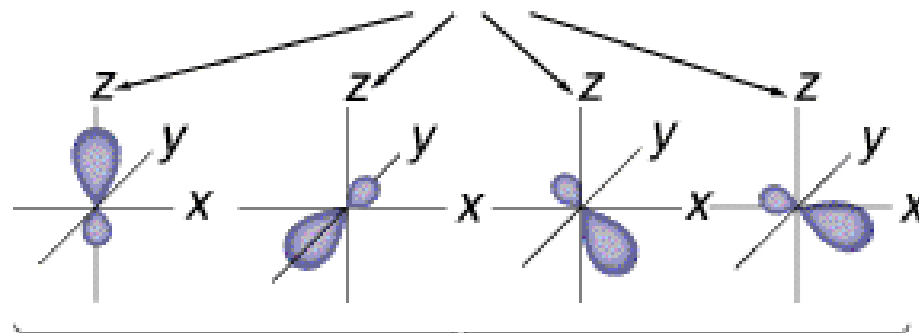
combine to generate
three sp^2 orbitals



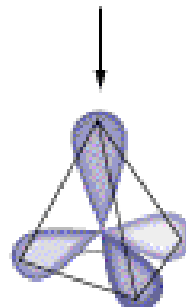
Formation of sp^3 hybrid orbitals



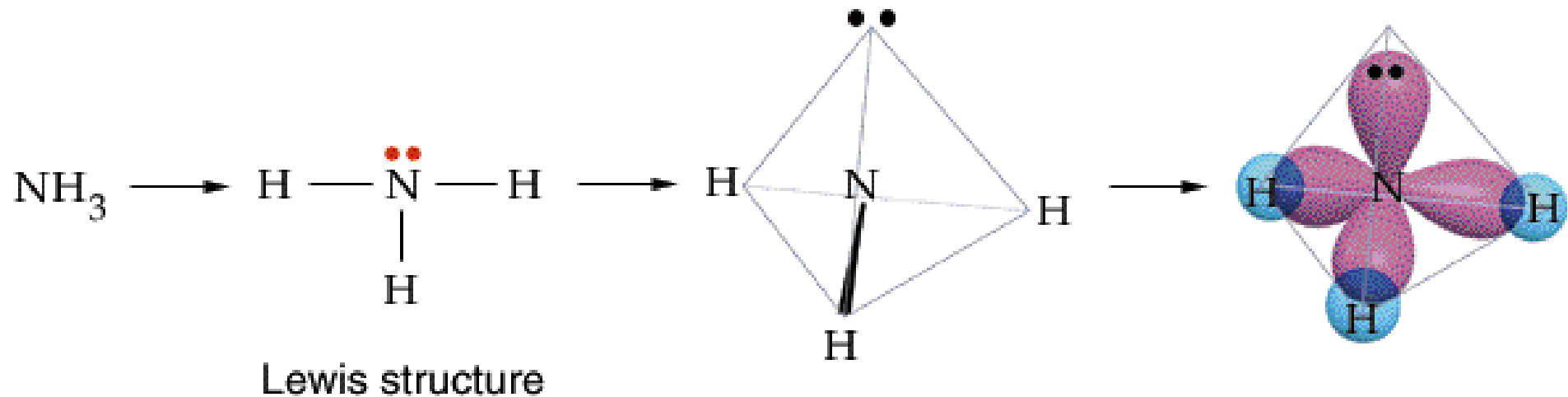
combine to generate
four sp^3 orbitals



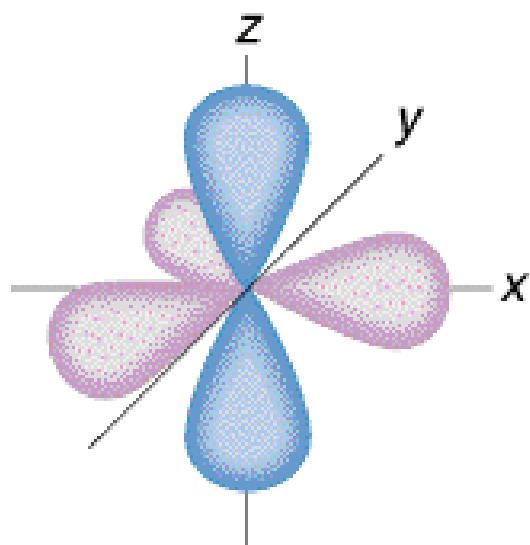
which are represented
as the set



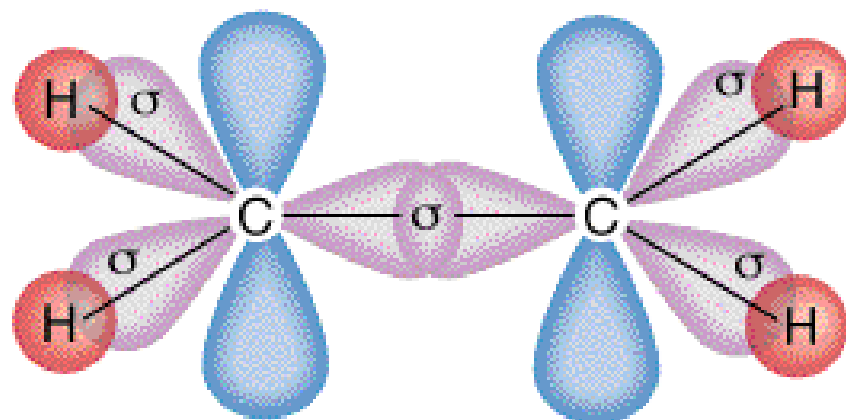
Hybrid orbitals can be used to explain bonding and molecular geometry



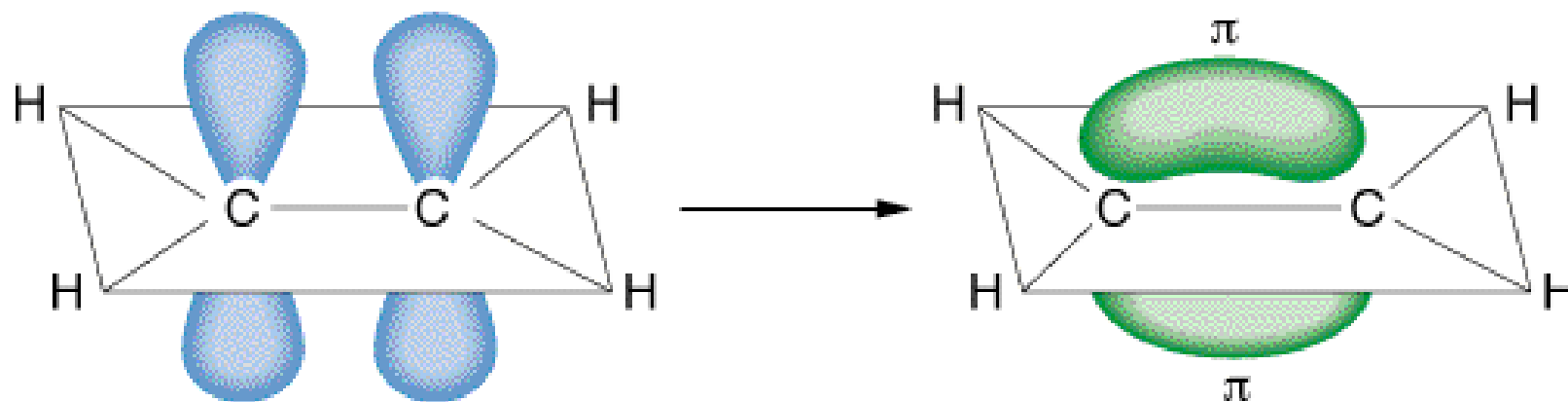
Example: $\text{H}_2\text{C}=\text{CH}_2$



the set of orbitals $sp^2 + p$

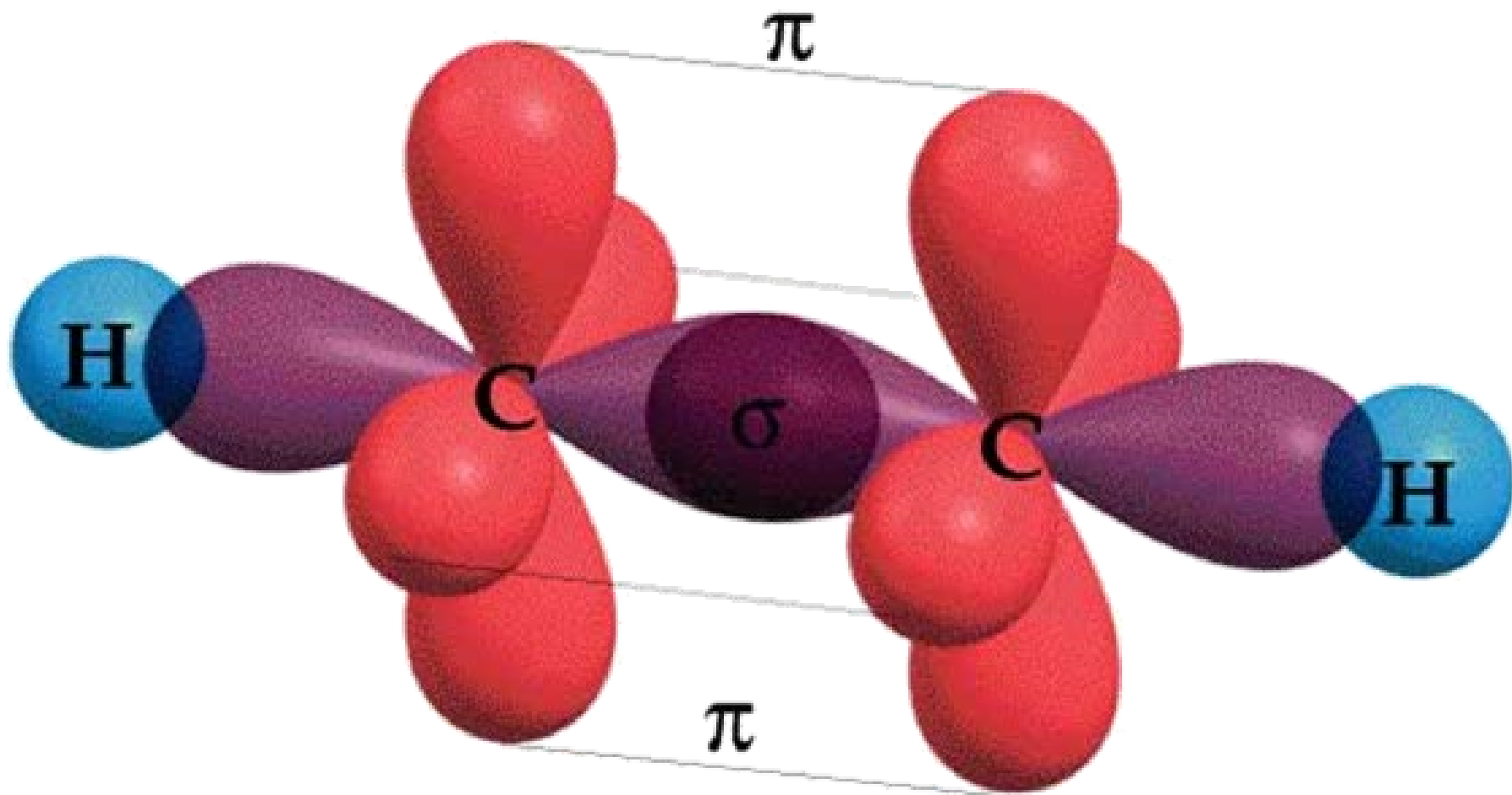


sigma (σ) bonds



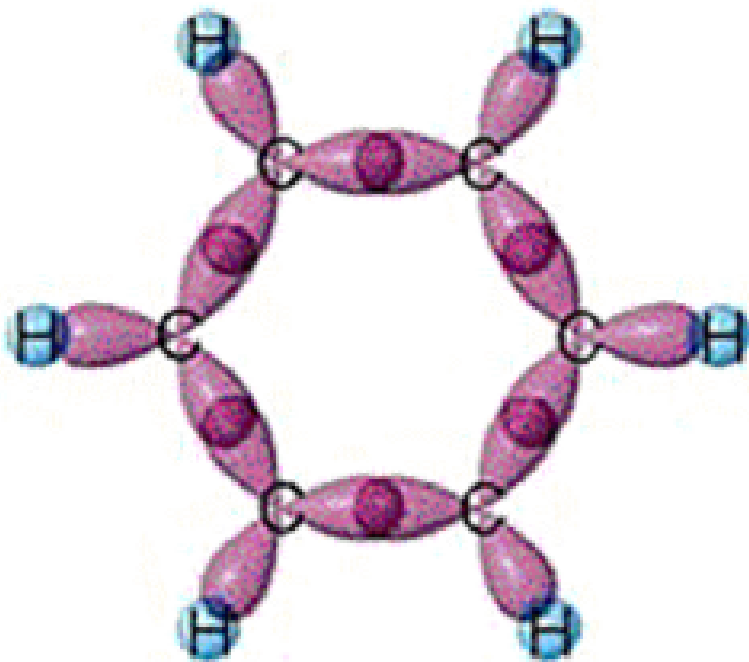
overlap of p orbitals leading to pi (π) bond

Example: $\text{HC}\equiv\text{CH}$

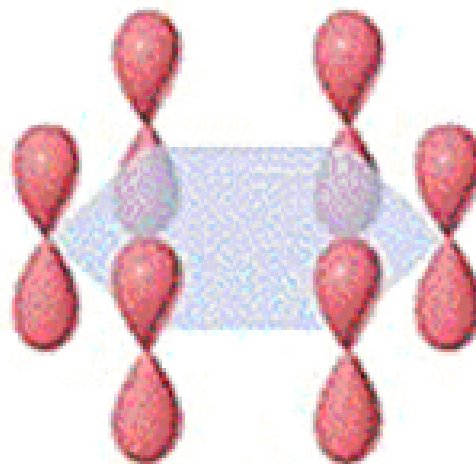


Delocalized π bonds

The pi electrons can be delocalized over all the atoms that have pi bond overlap.



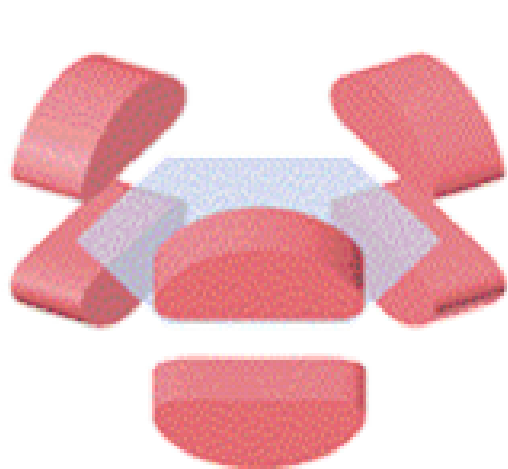
(a) σ bonds



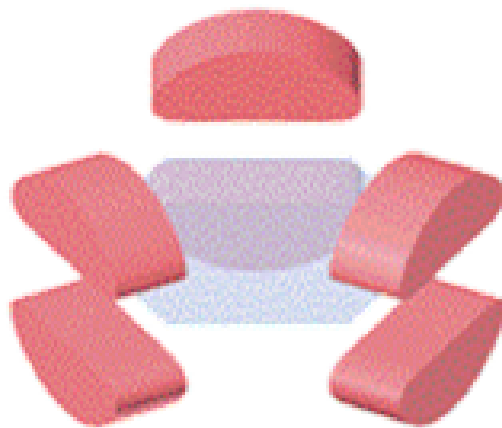
(b) 2p atomic orbitals

Example: C_6H_6 benzene

Benzene is an excellent example. For benzene the π orbitals all overlap leading to a very delocalized electron system



(a) Localized π bonds



(b) Localized π bonds



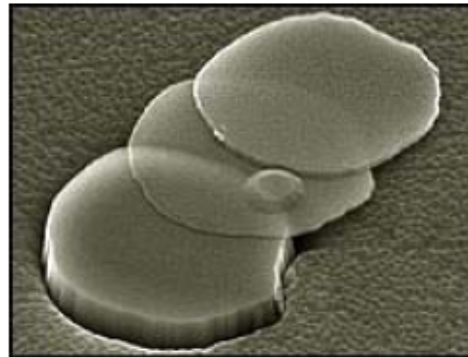
(c) Delocalized π bonds

Graphene discovery

Graphene, the world's first 2-dimensional fabric

Posted Oct 26, 2004, 3:30 PM ET

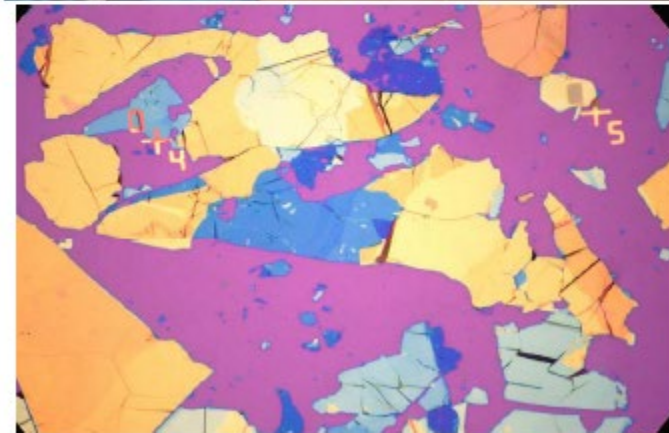
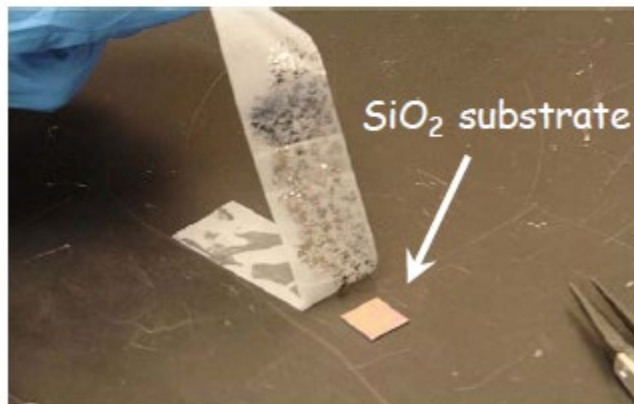
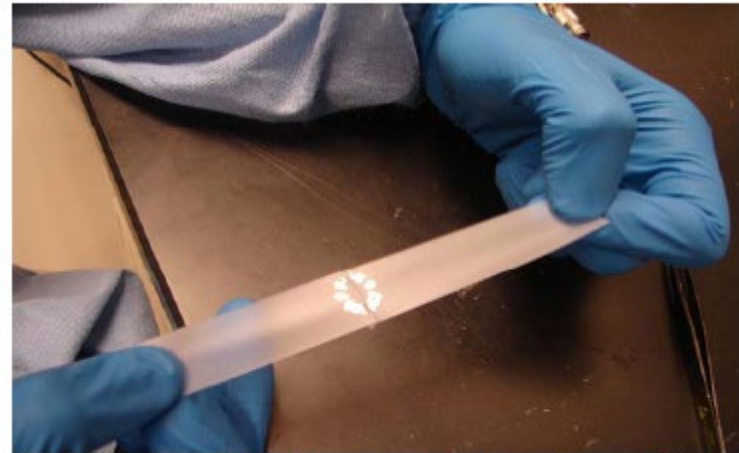
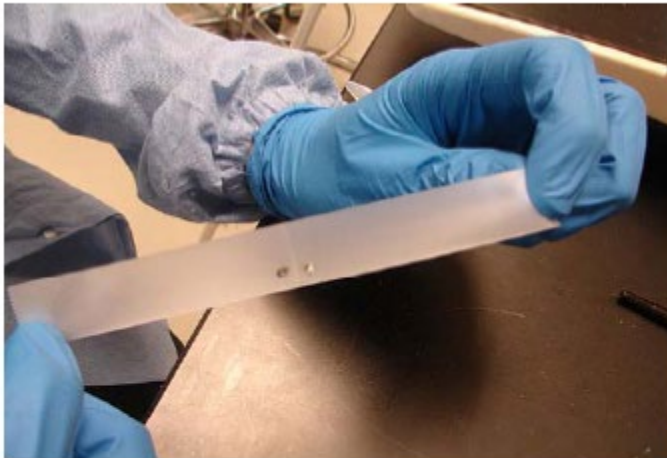
Researchers at The University of Manchester and Chernogolovka, Russia have created the first-ever single-atom-thick substance, a fabric they call “graphene”. The substance is stable, flexible, and highly conductive, and researchers believe it could be used to create computers made from a single molecule. Professor Andre Geim at The University of Manchester was able to extract a single plane of graphite crystal, resulting in the new fabric. The hope is that the fabric will be used in the future to create nanotubes, transistors for microscopic computers, that could result in some seriously small electronic gadgetry.



- high mobility (up to $\sim 200,000 \text{ cm}^2/(\text{Vs})$)
- transparent
- most importantly: opportunities for new effects and for bench-top experiments in quantum field theory

The Scotch trick

Novoselov et al., Science 2004



<http://www.sciam.com/article.cfm?id=diy-graphene-how-to-make-carbon-layers-with-sticky-tape>¹⁵

Short history

1564 Discovery of graphite (plumbago)

1779 Graphite is carbon

1789 Named from greek $\gamma\rho\alpha\varphi\epsilon\iota\nu$

Graphite: pencils, light bulbs, nuclear moderator,..

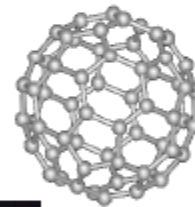
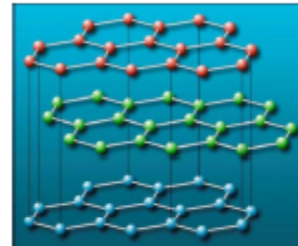
1962 HOPG (graphite monocrystal)

1960-1980 Graphite intercalation compounds

1985 Fullerenes [Kroto, Curl, Smalley]

1991-1993 Carbon nanotubes [Iijima]

1992-1993 few layers graphite on metal substrate



History: theory

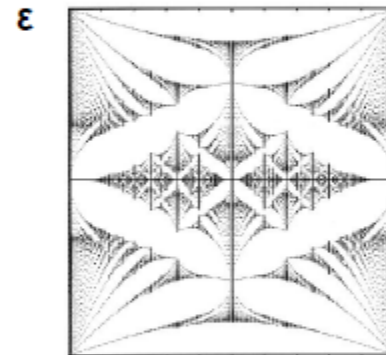
1947 Graphene band structure [Wallace]

1956 Graphene Landau levels [McClure]

1985 Hofstadter butterfly [Rammal]

1984-1988 Connection to 2+1 field theory [Semenoff, DiVincenzo & Φ/Φ_0
Mele, Fradkin, Haldane, etc.]

~90's Theory of carbon nanotubes
[Dresselhauss, Saito, Ando, etc.]



Graphene electronic structure

Graphene = 2D honeycomb carbon crystal

Carbon: 6 electrons $1s^2, 2s^2 2p^2$

hybridization: 1 orbital $2s$ and 2 orbitals $2p$

→ 3 orbitals sp^2

-3 coplanar σ bonds, angle 120° :

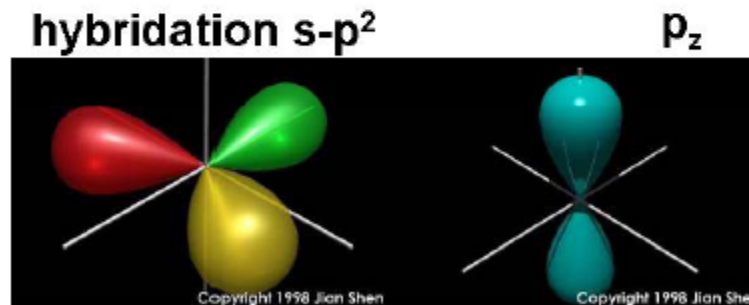
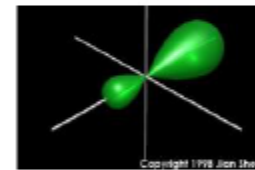
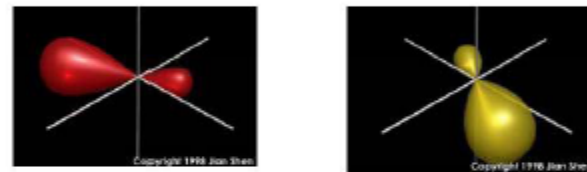
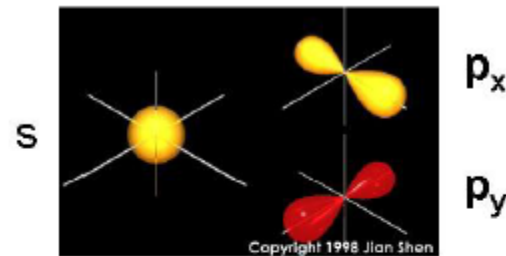
→ honeycomb structure

→ covalent bonding

+ 1 orbital $2p_z$ perpendicular to the plane

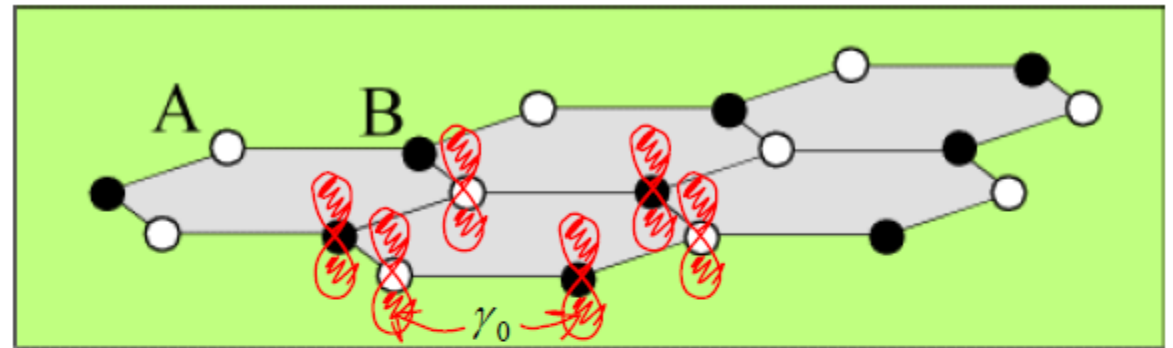
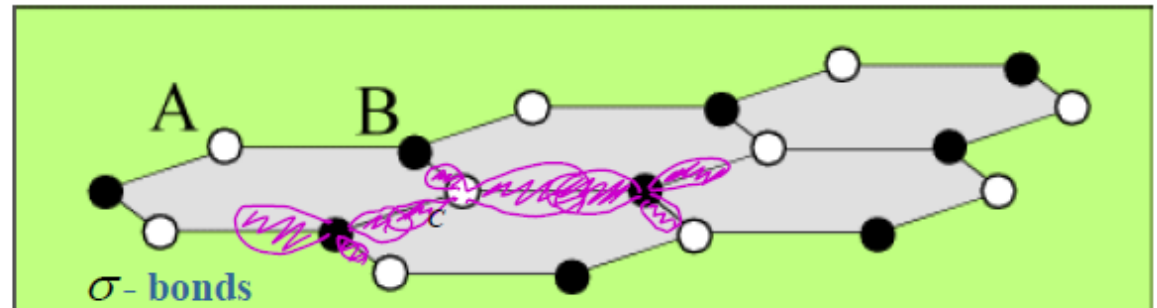
→ 1 conduction e per C

→ Half-filled band

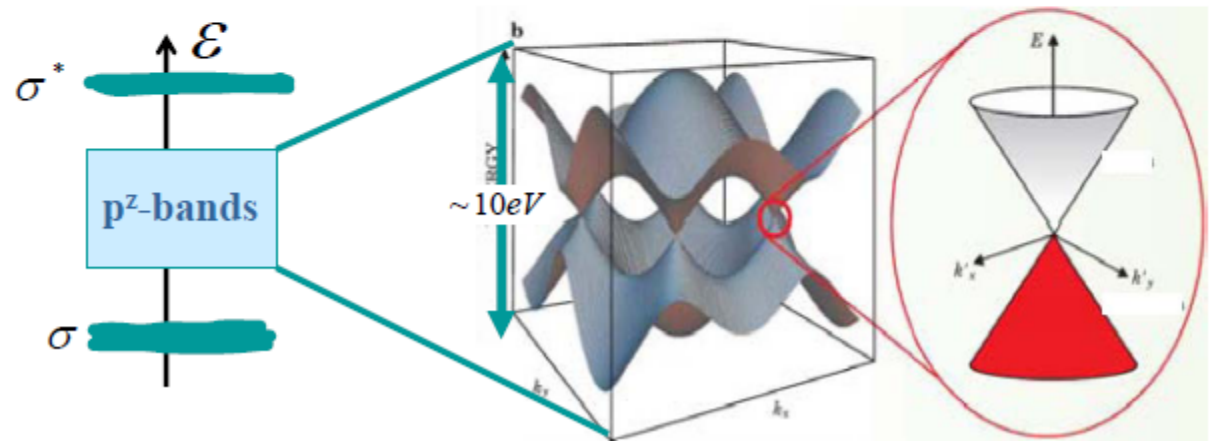


Carbon has 4 electrons in the outer s-p shell

sp^2 hybridisation forms strong directed bonds which determine a honeycomb lattice structure.

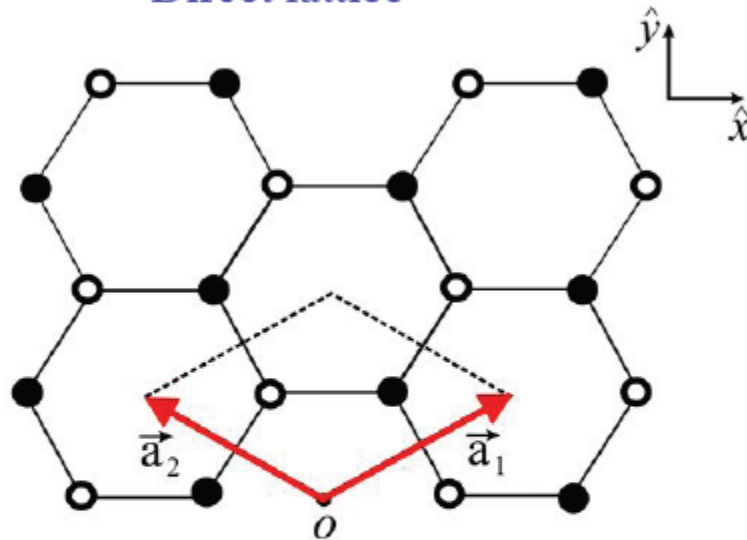


$p^z (\pi)$ orbitals determine conduction properties of graphite



2D Graphite (Graphene) unit cells

Direct lattice

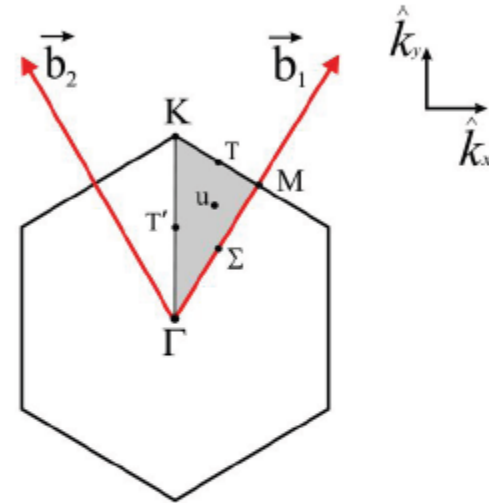


$$\vec{a}_1 = \frac{a}{2} (\sqrt{3} \hat{x} + \hat{y})$$

$$\vec{a}_2 = \frac{a}{2} (-\sqrt{3} \hat{x} + \hat{y})$$

$$a = 2.456 \text{ \AA}$$

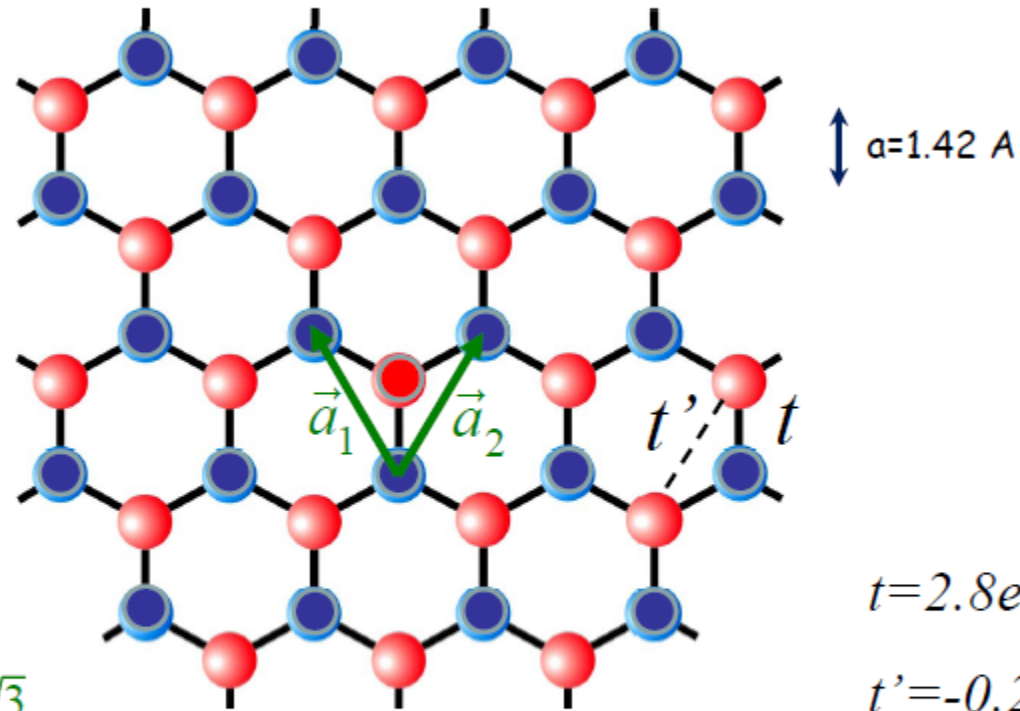
First Brillouin zone (BZ)



$$\vec{b}_1 = \frac{2\pi}{a} \left(\frac{\sqrt{3}}{3} \hat{k}_x + \hat{k}_y \right)$$

$$\vec{b}_2 = \frac{2\pi}{a} \left(-\frac{\sqrt{3}}{3} \hat{k}_x + \hat{k}_y \right)$$

« Honeycomb lattice » is not a Bravais lattice



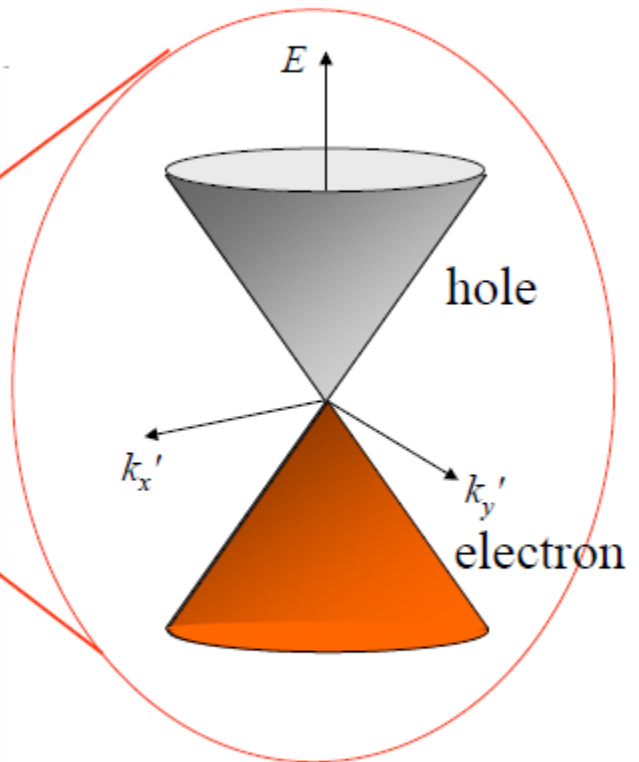
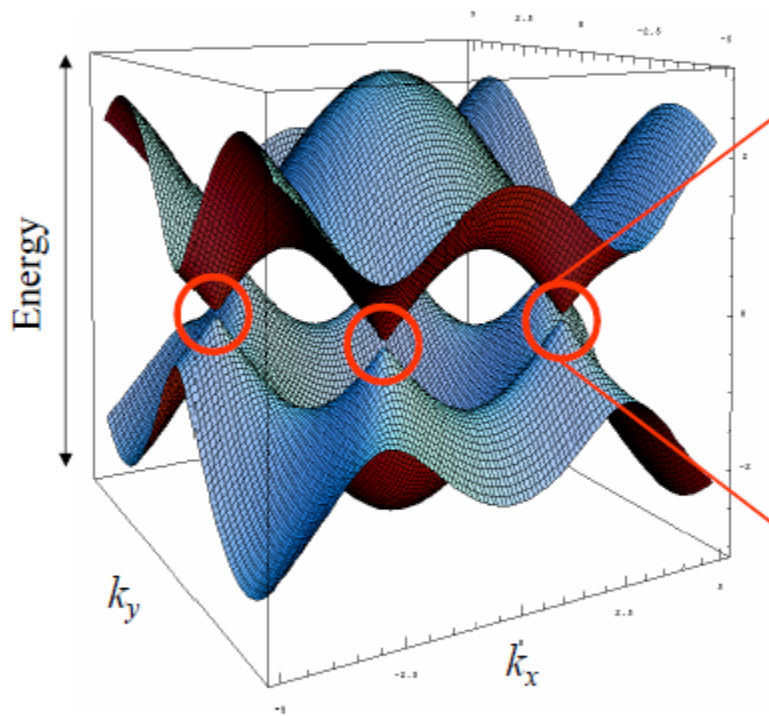
a = C-C distance
 a_0 = lattice parameter

Triangular Bravais lattice + 2 atoms per unit cell

$$E_k = \pm t \sqrt{1 \pm 4 \cos \frac{k_y a}{2} \cos \frac{\sqrt{3} k_x a}{2} + 4 \cos^2 \frac{k_y a}{2}}$$

Graphene : Dirac Particles in 2D Box

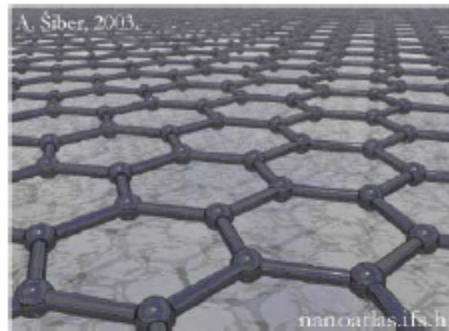
Band structure of graphene



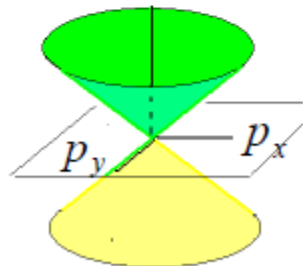
$$E \approx \hbar v_F |\vec{k}'_{\perp}|$$

Massless Dirac Particles with effective speed of light v_F

Graphene (monolayer of graphite)
is an atomically thin zero-gap
two-dimensional semiconductor
with linear dispersion of
conduction and valence band
electrons.



$$\varepsilon^{cond} = vp = v\sqrt{p_x^2 + p_y^2}$$



**Electronic dispersion in the vicinity of the corner of
the Brillouin zone: the same in both valleys.**

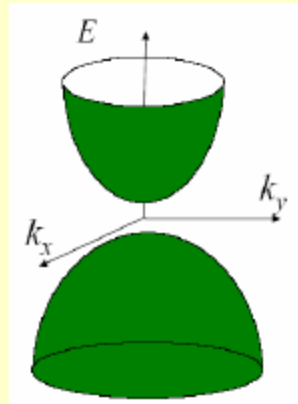
$$\varepsilon^{val} = -vp = -v\sqrt{p_x^2 + p_y^2}$$

Graphene v.s. Conventional 2D Electron System

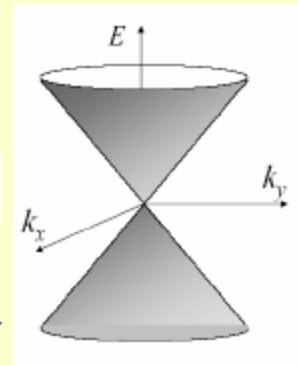
Conventional 2D Electron System

Band structures

$$E = \frac{\hbar^2 k^2}{2m_e^*}$$



$$E = -\frac{\hbar^2 k^2}{2|m_h^*|}$$

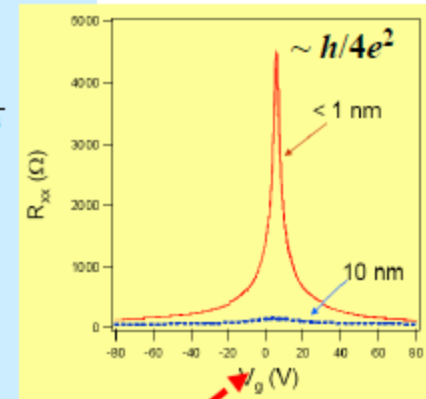
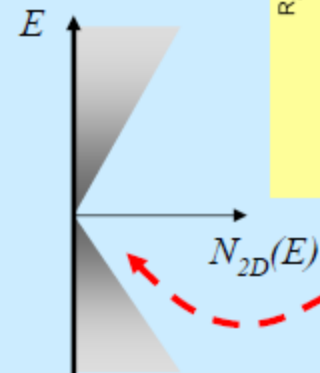
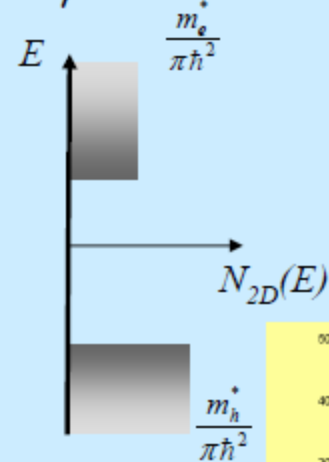


$$E = \hbar v_F |\vec{k}'|$$

Graphene

- Zero band mass
- Strict electron hole symmetry
- Electron hole degeneracy

Density of States



Science 306, 666 (2004)

Electric Field Effect in Atomically Thin Carbon Films

K. S. Novoselov,¹ A. K. Geim,^{1*} S. V. Morozov,² D. Jiang,¹
Y. Zhang,¹ S. V. Dubonos,² I. V. Grigorieva,¹ A. A. Firsov²

Vol 438|10 November 2005|doi:10.1038/nature04233

nature

LETTERS

Two-dimensional gas of massless Dirac fermions in graphene

K. S. Novoselov¹, A. K. Geim¹, S. V. Morozov², D. Jiang¹, M. I. Katsnelson³, I. V. Grigorieva¹, S. V. Dubonos²
& A. A. Firsov²

Vol 438|10 November 2005|doi:10.1038/nature04235

nature

LETTERS

Experimental observation of the quantum Hall effect and Berry's phase in graphene

Yuanbo Zhang¹, Yan-Wen Tan¹, Horst L. Stormer^{1,2} & Philip Kim¹

nature

Vol 462|12 November 2009|doi:10.1038/nature08582

LETTERS

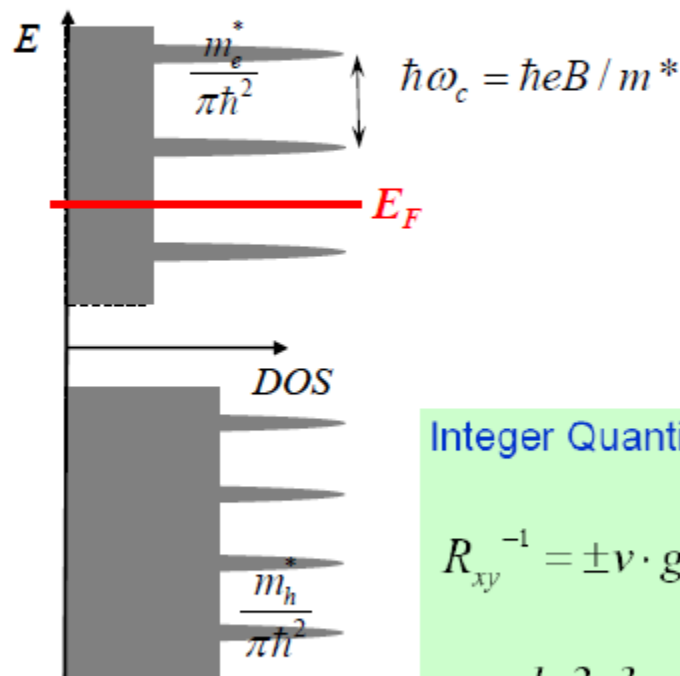
Observation of the fractional quantum Hall effect in graphene

Kirill I. Bolotin^{1*†}, Fereshte Ghahari^{1*}, Michael D. Shulman², Horst L. Stormer^{1,2} & Philip Kim^{1,2}

2D Gas in Quantum Limit : Conventional Case

Density of States

Landau Levels in Magnetic Field



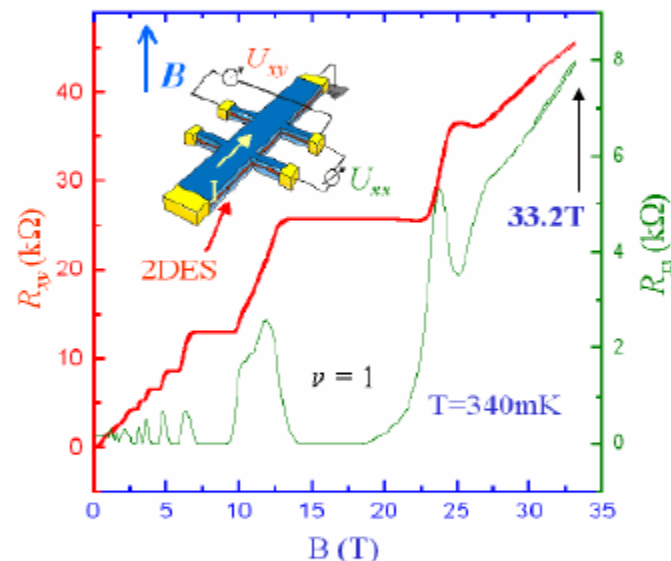
Integer Quantization:

$$R_{xy}^{-1} = \pm \nu \cdot g_s \cdot \frac{e^2}{h}$$

$$\nu = 1, 2, 3 \dots$$

$$g_s = 2 \text{ (spin)}$$

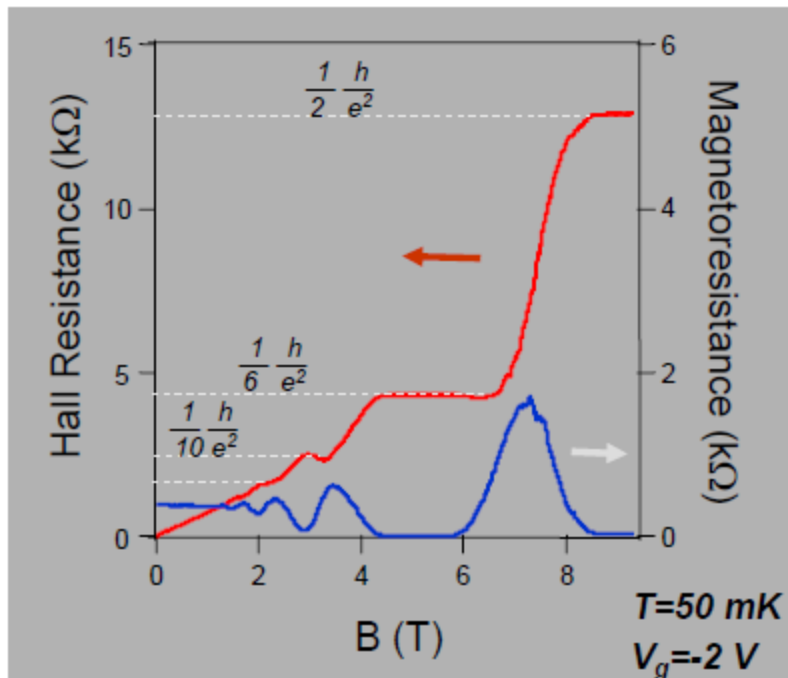
Quantum Hall Effect in GaAs 2DEG



Graphene

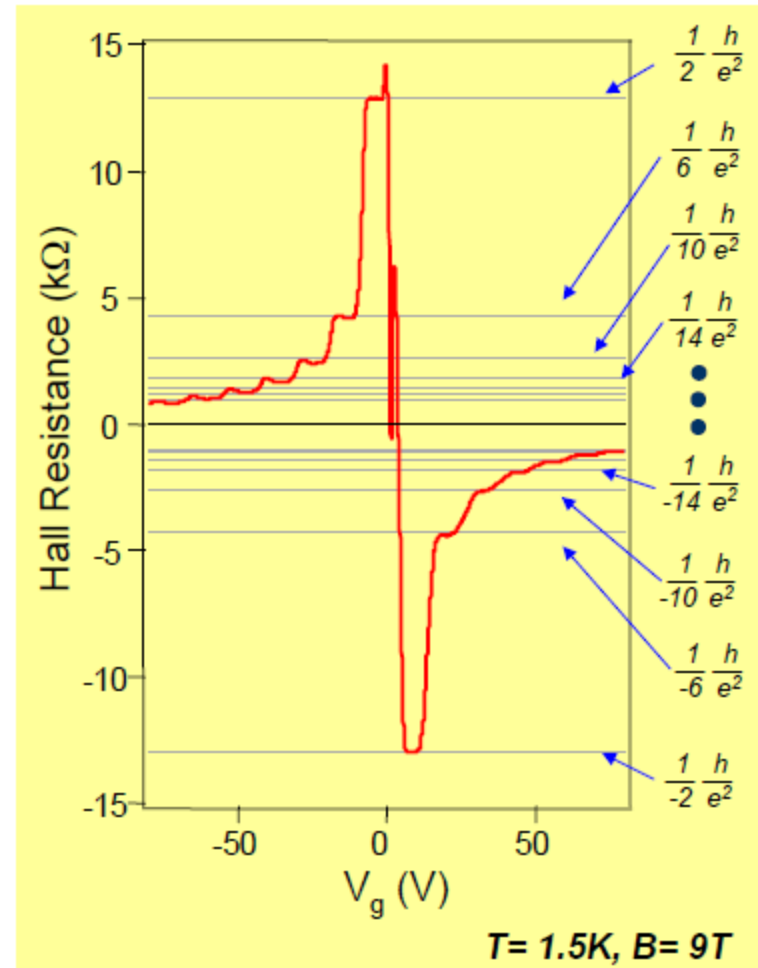
- Vanishing carrier mass near Dirac point
- Strict electron hole symmetry
- Electron hole degeneracy $\omega_c = \frac{eB}{m^*}$

Quantum Hall Effect in Graphene



Quantization:

$$R_{xy}^{-1} = 4 \left(n + \frac{1}{2} \right) \frac{e^2}{h}$$



Relativistic Landau Level and Half Integer QHE

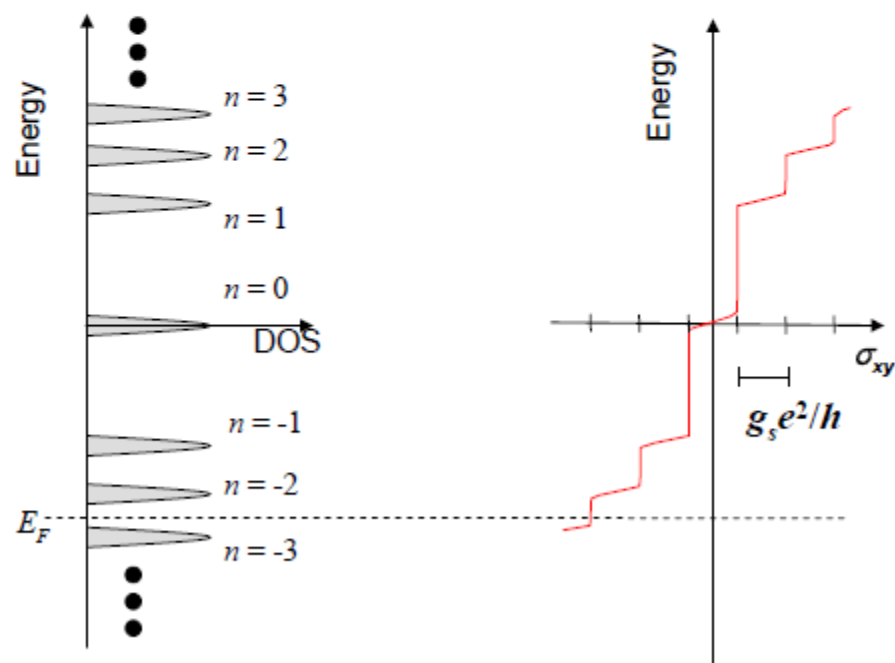
Haldane, PRL (1988)

Landau Level $E_n = \pm \sqrt{2e\hbar v_F^2 |n| B}$

Landau Level Degeneracy

$$g_s = 4$$

2 for spin and 2 for sublattice



Quantized Condition

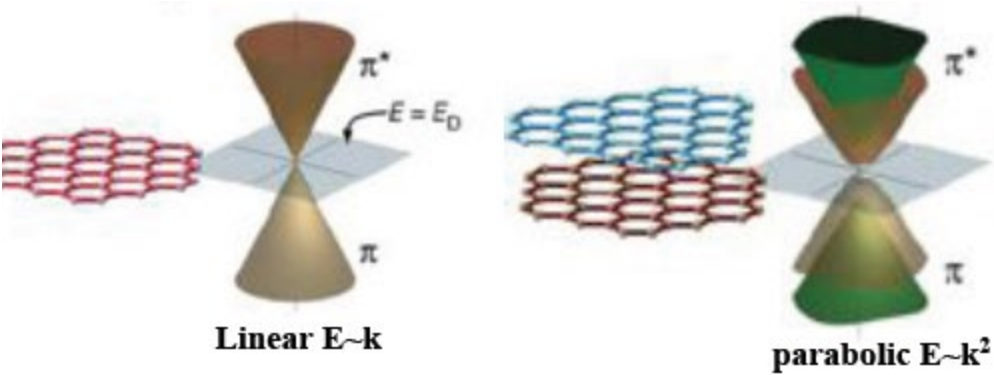
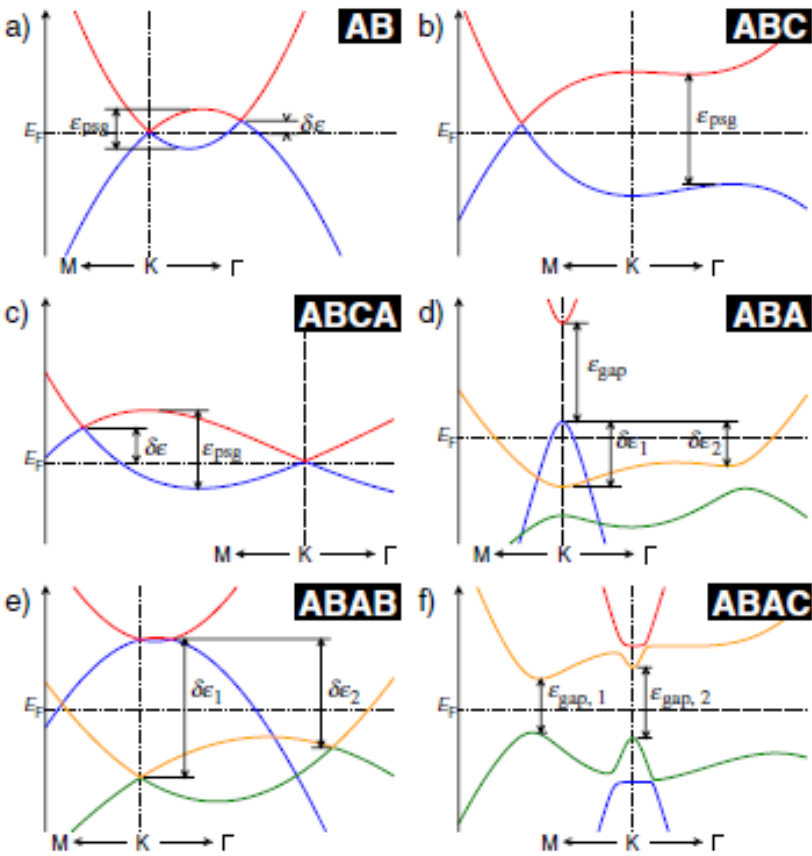
$$R_{xy}^{-1} = \pm g_s \left(n + \frac{1}{2}\right) \frac{e^2}{h}$$

$$\nu = \pm g_s \left(n + \frac{1}{2}\right)$$

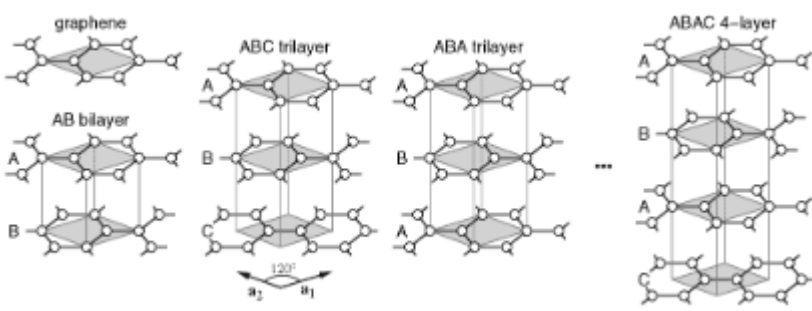
T. Ando et al (2002)

Charge Carriers in Few-Layer Graphene Films

Sylvain Latil and Luc Henrard

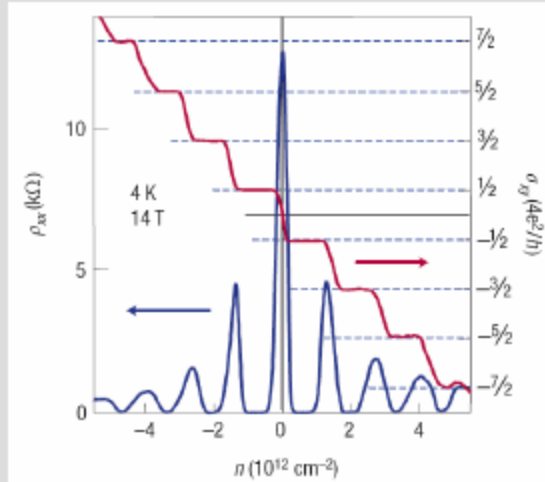


Taisuke Ohta, *et al.*, Science **313**, 951 (2006)



Geometry of stacking and the number of layers strongly affect band structure

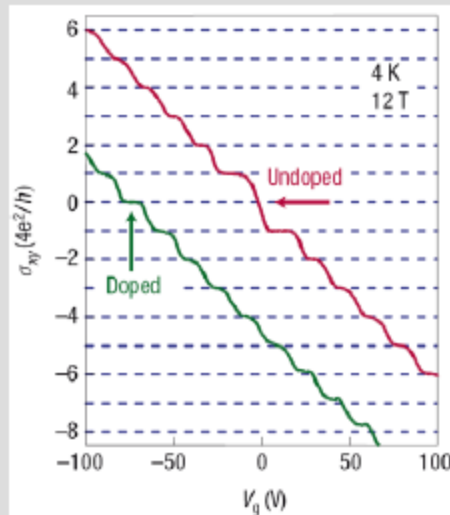
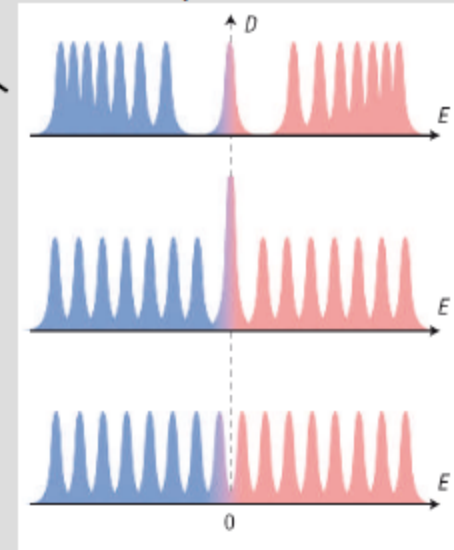
half-integer QHE



graphene:
half-integer QHE
no plateau at zero energy

$$\sigma_{xy} = \pm 4e^2/h (N + 1/2)$$

Landau levels in the density of states



bilayer graphene:
integer QHE

but

N=0 plateau missing!

$$\sigma_{xy} = \pm N 4e^2/h$$