PH 673 Nano

Lecture # 7

(1)

Photonic crystals

Consider Maxwell's equations ⇒

$$\vec{\nabla}\times\vec{E}(\vec{r},t) = -\mu_0 \frac{\partial \vec{H}(\vec{r},t)}{\partial t} \; ; \quad \vec{\nabla}\times\vec{H}(\vec{r},t) = \varepsilon_0 \varepsilon(\vec{r}) \frac{\partial \vec{E}(\vec{r},t)}{\partial t}$$

$$\vec{\nabla}\cdot[\varepsilon(\vec{r})\vec{E}(\vec{r},t)] = 0 \; ; \quad \vec{\nabla}\cdot\vec{H}(\vec{r},t) = 0$$

Harmonic modes ⇒ $\begin{matrix}\vec{H}(\vec{r},t) \\ \vec{E}(\vec{r},t)\end{matrix} = \begin{matrix}\vec{H}(\vec{r}) \\ \vec{E}(\vec{r})\end{matrix} e^{-i\omega t}$

⇓

$$\vec{\nabla}\times\vec{E}(\vec{r}) - i\omega\mu_0\vec{H}(\vec{r}) = 0 \; ; \quad \vec{\nabla}\times\vec{H}(\vec{r}) + i\omega\varepsilon_0\varepsilon(\vec{r})\vec{E}(\vec{r}) = 0$$

⇙ divide by $\varepsilon(\vec{r})$ and take curl

$$\vec{\nabla}\times\left(\frac{1}{\varepsilon(\vec{r})}\vec{\nabla}\times\vec{H}(\vec{r})\right) + i\omega\varepsilon_0 \underbrace{\vec{\nabla}\times\vec{E}(\vec{r})}_{i\omega\mu_0\vec{H}(\vec{r})} = 0 \Rightarrow \boxed{\vec{\nabla}\times\left(\frac{1}{\varepsilon(\vec{r})}\vec{\nabla}\times\vec{H}(\vec{r})\right) = \left(\frac{\omega}{c}\right)^2\vec{H}(\vec{r})}$$

$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

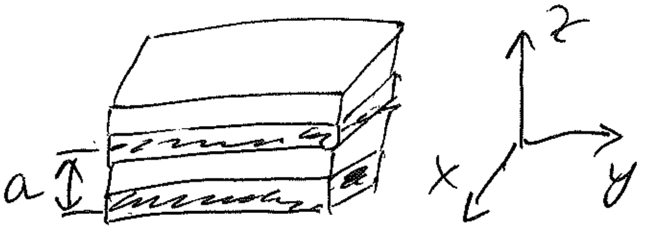
⇙ solve this subject to $\vec{\nabla}\cdot\vec{H}(\vec{r}) = 0$

and then, $\vec{E}(\vec{r}) = \frac{i}{\omega\varepsilon_0\varepsilon(\vec{r})}\vec{\nabla}\times\vec{H}(\vec{r})$ ⇐ this will automatically satisfy $\vec{\nabla}\cdot[\varepsilon(\vec{r})\vec{E}(\vec{r})] = 0$

Introduce Maxwell operator $\hat{\Theta} = \vec{\nabla}\times\left(\frac{1}{\varepsilon(\vec{r})}\vec{\nabla}\times\right)$ ⇒

$\hat{\Theta}\vec{H}(\vec{r}) = \left(\frac{\omega}{c}\right)^2\vec{H}(\vec{r})$ – eigenvalue problem! \ linear, Hermitian

Consider 1D photonic crystal $\Rightarrow$ multilayer film



$\varepsilon(z)$

in x-y plane

Modes $\Rightarrow$ $\vec{H}_{n,K_z,\vec{K}_{\parallel}}(\vec{r}) = e^{i\vec{K}_{\parallel}\cdot\vec{\rho}}\, e^{iK_z z}\, \vec{u}_{n,K_z,\vec{K}_{\parallel}}(z)$

$\vec{u}(z) = \vec{u}(z+R)$, $R = na$

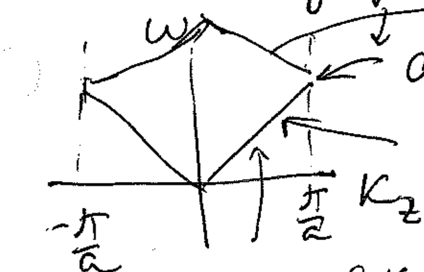
$\vec{K}_{\parallel}$ - any value, K_z restricted to $-\frac{\pi}{a} < K_z \leq \frac{\pi}{a}$

$\Downarrow$

photonic band gap!

air band

depends on the contrast in $\varepsilon(z)$



dielectric band

$\frac{\Delta\omega \leftarrow \text{gap}}{\omega_m} \approx \frac{\Delta\varepsilon}{\varepsilon} \frac{\sin\frac{\pi d}{a}}{\pi}$

Specific case: $\omega = \frac{cK}{\sqrt{\varepsilon}}$

$\frac{\lambda}{4}$ - stack $\Rightarrow$ $\lambda_m = \frac{2\pi c}{\omega_m}$ $\leftarrow$ midgap frequency

such that

(normal incidence) $\frac{\lambda_m}{4} = n_1 d_1 = n_2 d_2$; $(d_1 + d_2) = a$; $d_1 = \frac{a n_2}{n_1 + n_2}$

refractive indices $\Rightarrow \omega_m = \frac{n_1 + n_2}{4 n_1 n_2} \frac{2\pi c}{a}$

$\Downarrow$

reflected waves from each layer are in phase at the ω_m $\Rightarrow$ minimal penetration of light into the crystal

$\frac{\Delta\omega}{\omega_m} = \frac{4}{\pi} \sin^{-1} \frac{|n_1 - n_2|}{n_1 + n_2} = \frac{4}{\pi} \sin^{-1} r$;

r — reflection coeff.

QM in a periodic potential (crystal)	E&M in a periodic dielectric (photonic crystal)
$\Psi(\vec{r},t)$	$\vec{H}(\vec{r},t)$ ← magnetic field
$\Psi(\vec{r},t)=\sum_{E} C_E \Psi_E(\vec{r}) \cdot e^{-i\frac{Et}{\hbar}}$ expansion in energy eigenstates	$\vec{H}(\vec{r},t)=\sum_{\omega} C_\omega H_\omega(\vec{r}) e^{-i\omega t}$ expansion in a set of harmonic modes (frequency eigenstates)
Schrödinger equation ⇒ $\left[-\frac{\hbar^2}{2M}\vec{\nabla}^2+V(\vec{r})\right]\Psi_E(\vec{r})= =E\Psi_E(\vec{r})$	Maxwell equations ⇒ $\underbrace{\vec{\nabla}\times\frac{1}{\varepsilon(\vec{r})}\nabla\times}_{\hat{\Theta}}\vec{H}_\omega(\vec{r})=\frac{\omega^2}{c^2}\vec{H}_\omega(\vec{r})$ $\hat{\Theta}$ ← Maxwell operator
$V(\vec{r})=V(\vec{r}+\vec{R})$ lattice vectors	$\varepsilon(\vec{r})=\varepsilon(\vec{r}+\vec{R})$ lattice vectors
Ψ_E's for different E are orthogonal, have real eigenvalues	Modes with different frequencies are orthogonal, have ≥0 real values
$\hat{H}$ is a linear Hermitian operator	$\hat{\Theta}$ is a linear Hermitian operator
$E_{var}=\frac{\langle\Psi,\hat{H}\Psi\rangle}{\langle\Psi\vert\Psi\rangle}$ is minimized when Ψ is an eigenstate of $\hat{H}$	$\nu_{var}=\frac{(\vec{H},\hat{\Theta}\vec{H})}{(\vec{H},\vec{H})}$ is minimized when $\vec{H}$ is an eigenstate of $\hat{\Theta}$

energy: eigenvalue E of $\hat{H}$

$$U = \frac{1}{4}\int d^3\vec{r}\,(\varepsilon_0 \varepsilon |\vec{E}|^2 + \mu_0 |\vec{H}|^2)$$

$[\hat{A}, \hat{H}] = 0 \Rightarrow \hat{A}$ is a symmetry of the system

$[\hat{A}, \hat{\Theta}] = 0$

translational symmetry

$\Psi_{\vec{k}}(\vec{r}) = u_{\vec{k}}(\vec{r})\, e^{i\vec{k}\cdot\vec{r}}$

$\vec{H}_{\vec{k}}(\vec{r}) = \vec{u}_k(\vec{r})\, e^{i\vec{k}\cdot\vec{r}}$

"band structure" $\Rightarrow E_n(\vec{k})$

energies of eigenstates

$\omega_n(\vec{k}) \leftarrow$ frequencies of harmonic modes

"gap" in the band structure $\Rightarrow$ within that range of energies, there are no propagating electron states, regardless of k

$\Rightarrow$ no propagating EM modes regardless of k or polarization

conduction / valence bands

air / dielectric bands

defects: foreign atoms (break translational symmetry)

$\Rightarrow$ change $\varepsilon(\vec{r})$ at some locations

result of a defect: may create an allowed state in the gap $\Rightarrow$ localized electron

$\Rightarrow$ localized EM mode

donor / acceptor

dielectric / air defects

overall: tailor electronic properties

tailor optical properties