

Fractional Charge

Particles with charge $e/3$ and $e/5$ have been observed experimentally...

...and they're not quarks.

Outline:

1. What is fractional charge?
2. Observing fractional charge in the fractional quantum Hall effect
3. New theoretical (and hopefully experimental) directions

old work with A. Ludwig and H. Saleur

and with K. Intriligator

new work with R. Moessner and S. Sondhi

and with K. Schoutens, B. Nienhuis and J. de Boer

Important principle:

All of condensed-matter and particle (and now, some of AMO) physics is many-body physics.

In particular, the ground state of a quantum system is not a “vacuum”.

In condensed-matter physics, we have the **Fermi sea**.

In particle physics, the ground state is full of **virtual particles**, which can pop in and out.

As a result, many counterintuitive effects occur in nature.

excitation = quasiparticle = particle

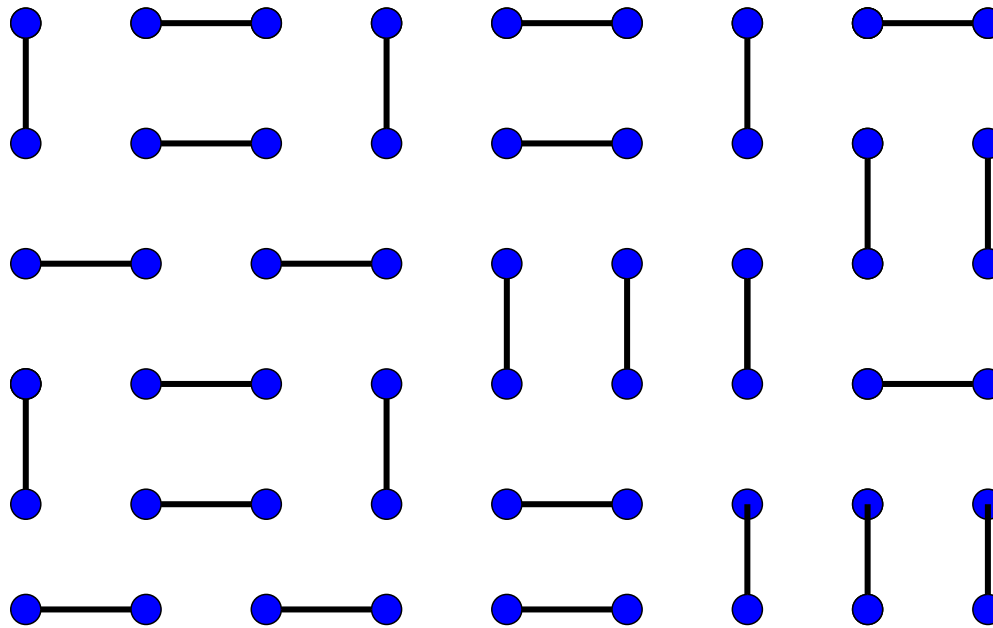
These words all mean an eigenstate of the quantum Hamiltonian with localized momentum and energy relative to the ground state

One might expect that the quasiparticles over a Fermi sea have quantum numbers (charge, spin) of an electron.

Not necessarily true!

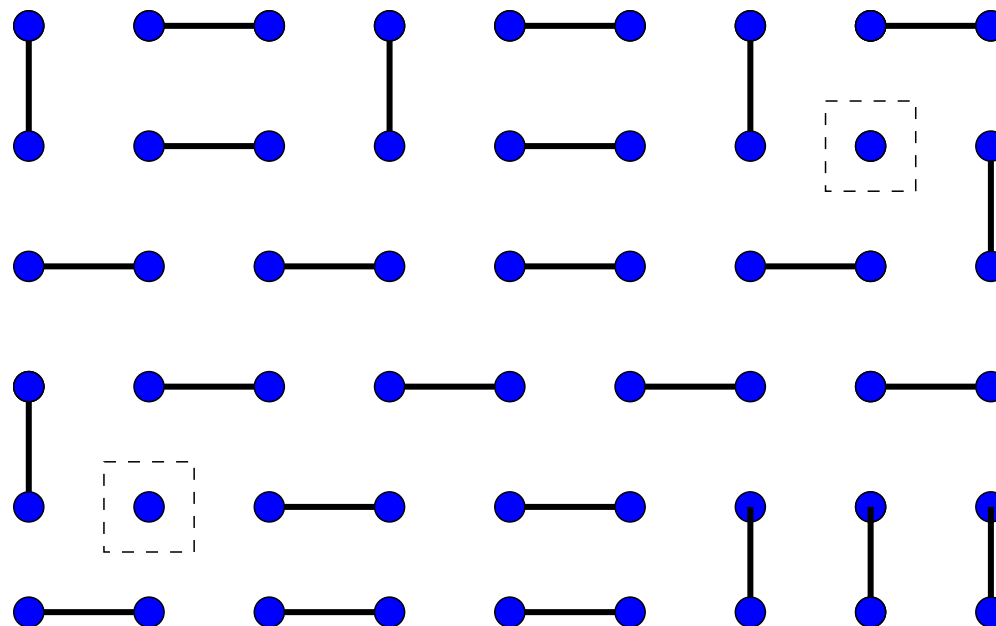
“RVB” phase in 2d

Electrons (spin $1/2$, charge e) on a square lattice, one per site. Say the interactions favor a spin singlet on neighboring sites. The ground state will be a superposition of “dimer” states like



Each dimer has spin 0 and charge $2e$, so the ground state has spin 0 and charge $N e$ for N sites. This is called a **resonating valence bond** state.

To get excited state, break **one** bond



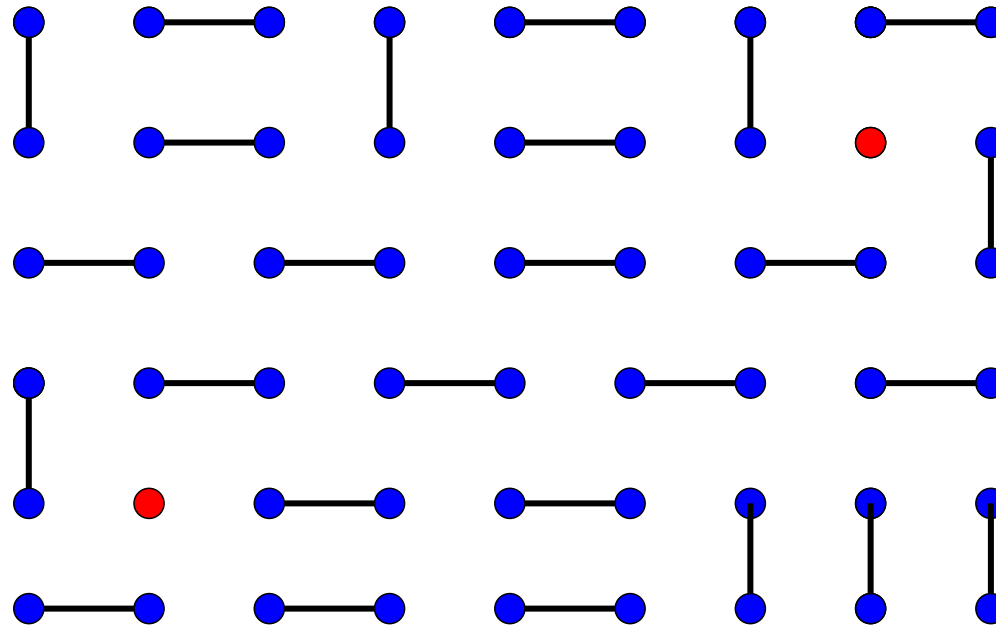
If there is no confinement, you make **two** quasiparticles. This state has the same charge as the ground state, but different spin.

Thus these particles have **charge 0** and **spin $1/2$** !!

They are called **spinons**.

Anderson; Kivelson, Rokhsar and Sethna

Say you remove two electrons, and create a defect:



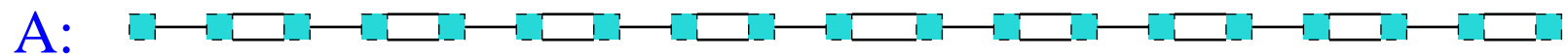
This two-hole state has no spin, and charge $-2e$ relative to the ground state. Each quasiparticle has **charge $-e$** and **spin 0** !!

This is like **oblique confinement** in QCD at $\theta = \pi$: quarks are free, but have no color.

The electron has “split” into a **spinon** and a **holon**! This is called **spin-charge separation**.

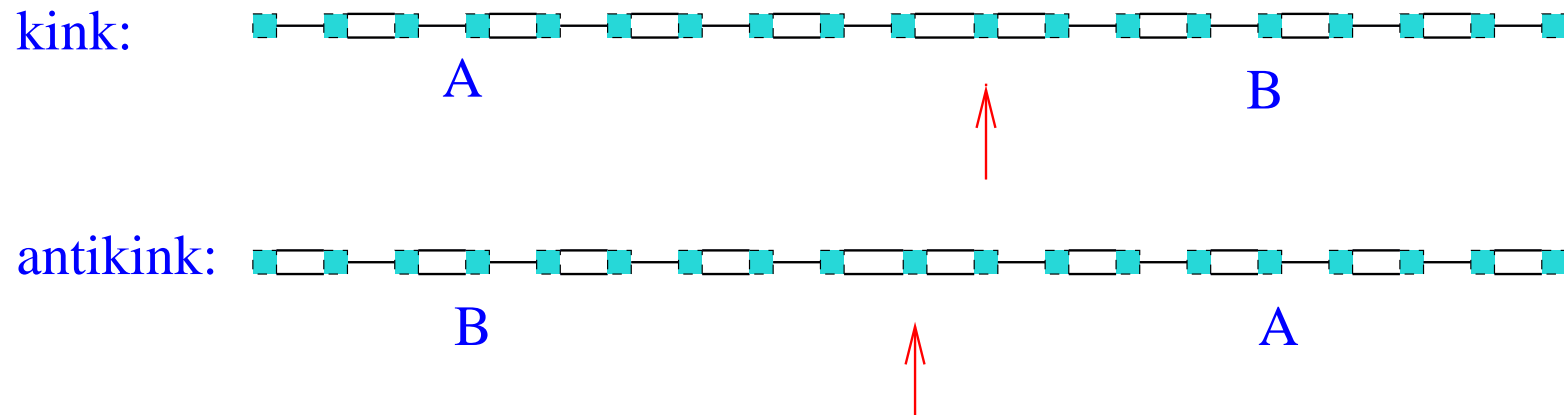
We can go further.

Polyacetylene is a conducting polymer (i.e. it is a **1d chain with mobile electrons**). It is energetically favorable for it to alternate between **double** (two shared electrons) and **single** (one shared electron) bonds. Thus there are **two ground states**:



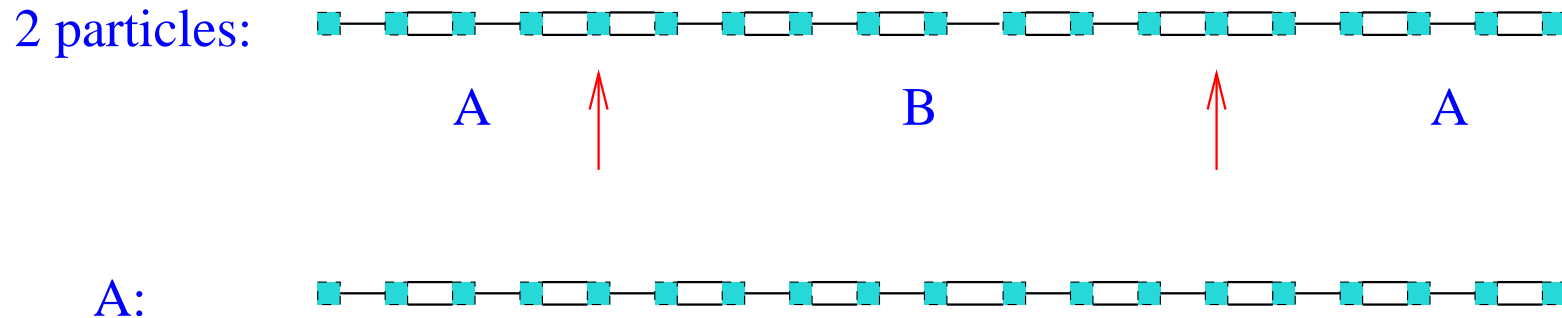
Note the electrons now are the bonds, not the sites.

Defects in this pattern (two double bonds in a row, or two single bonds) are quasiparticles. Defects in 1d are often called kinks. For an extra double bond:



Likewise, there is a kink and an antikink with one extra single bond.

Now, the amazing part. Look at the state with two double-bond quasiparticles:



The state has just one extra electron! (28 vs. 27 in the picture)

You can move these two quasiparticles arbitrarily far apart. They are identical. The energy is localized. The only possible conclusion:

Each quasiparticle has charge $e/2$!

A two-particle state with one less electron has two charge $-e/2$ kinks.

Two questions:

Can we find theories (i.e. actual Hamiltonians) with such behavior?

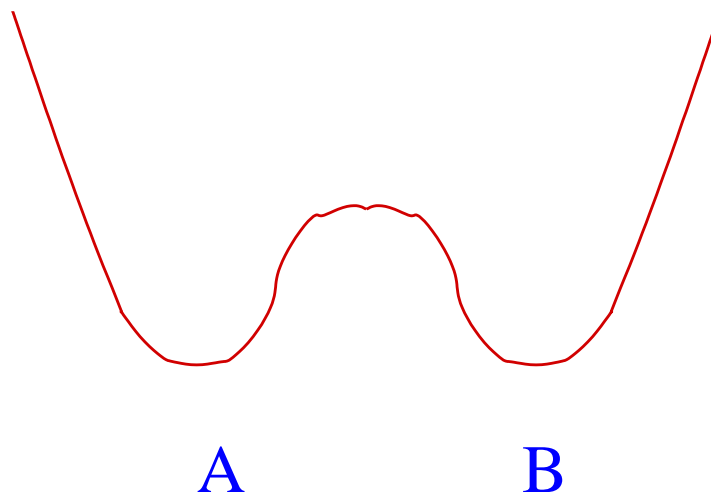
Are these theories be realized in nature?

One answer: **Yes!**

In fact, this happens in a field theory of a single boson coupled to a Dirac fermion in $1 + 1$ dimensions. Take the potential for the boson ϕ to be

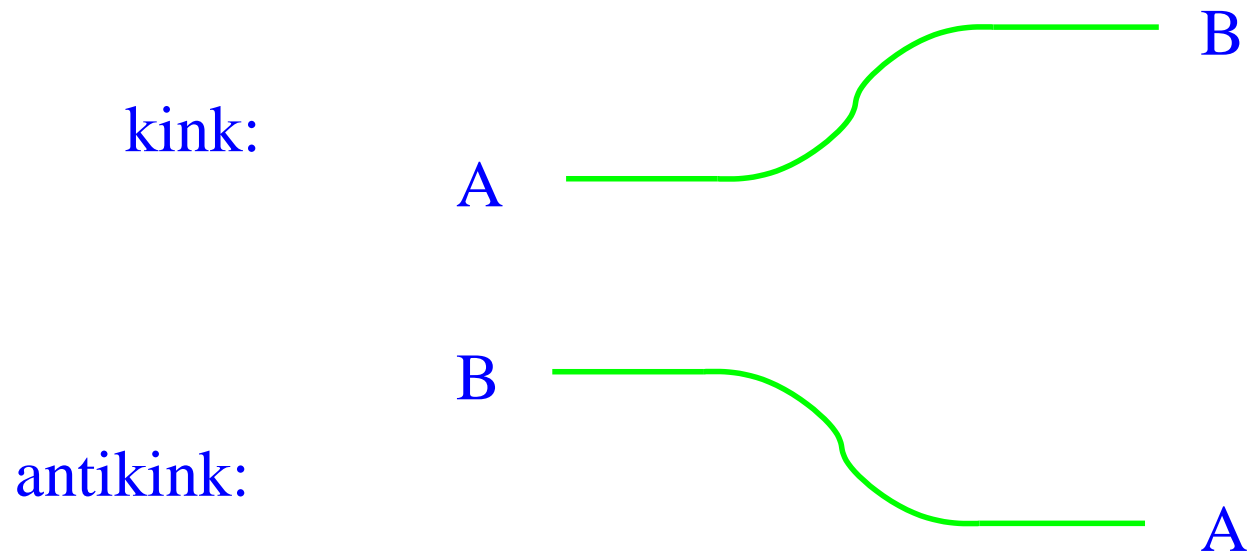
$$V(\phi) = (\phi^2 - \lambda)^2$$

(just plain old ϕ^4 theory with spontaneous symmetry-breaking).



The potential has two minima, at $\phi = \pm\sqrt{\lambda}$. Label these vacua A and B . A field configuration with $\phi(x = -\infty) = -\sqrt{\lambda}$ and $\phi(x = \infty) = \sqrt{\lambda}$ is a kink. The reverse is an antikink.

The field configurations $\phi(x)$ for a kink and antikink look like



Just like in polyacetylene!

Now solve the Dirac equation in the background of a kink. You find that there are **two** solutions with minimum energy, which

- differ in charge by e .
- are related by charge-conjugation invariance.

(There is a single fermion zero mode, which can be either filled or unfilled.)

Only possibility consistent with both criteria is that the two states have charge $+e/2$ and $-e/2$!!

field theory: Jackiw and Rebbi, 1976

polyacetylene: Su, Schrieffer and Heeger, 1979

Experimental data on polyacetylene is consistent with the existence of fractional charge, but not conclusive.

One can find $1 + 1$ -dimensional field theories with arbitrary fractions of e .

Goldstone and Wilczek, 1981

One can “prove” the existence of arbitrary rational fractional charge in $1 + 1$ dimensions by solving certain supersymmetric field theories/lattice models.

Fendley and Intriligator, 1991

Fendley, Nienhuis and Schoutens, 2003

To unambiguously observe fractional charge, one must go to the...

Fractional quantum Hall effect

Trap electrons in two dimensions. Apply a very strong transverse magnetic field. Apply a voltage. Current flows perpendicular to the voltage.

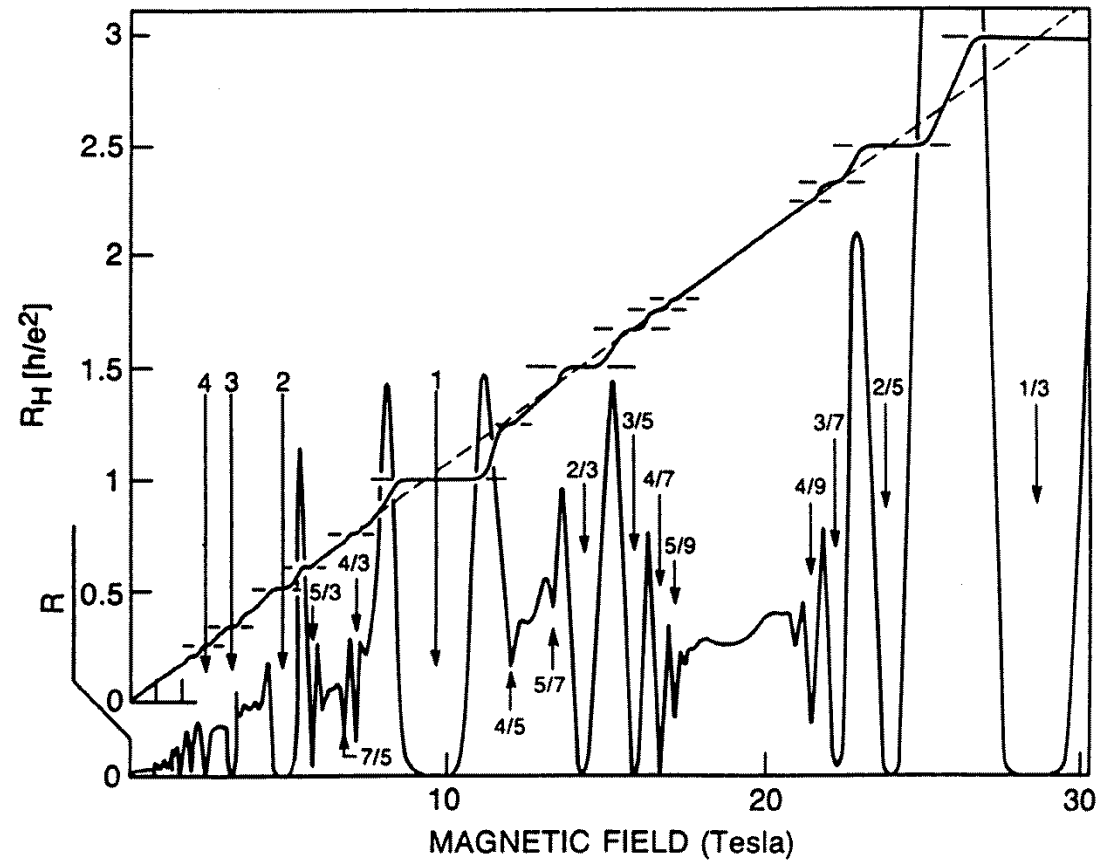
Classically,

$$R_{Hall} = \frac{V_y}{I_x}$$

is linear in the magnetic field:

$$R_{Hall} \propto H$$

But at high enough magnetic fields...



data from H. Stormer, *Physica B*177 (1992) 401

There are very flat plateaus in R_{Hall} at a rational number times h/e^2 (with precision a part in 10^8). On these plateaus, R_{yy} drops to zero.

Write $G_{Hall} = \nu e^2 / h$. The integer QHE describes those plateaus where ν is an integer. These are fairly well-understood theoretically: they are related to the effect of disorder on Landau levels.

The fractional QHE describes the plateaus where ν is a fraction, the most prominent occurring at $\nu = 1/3$.

Tsui, Stormer and Gossard, 1982

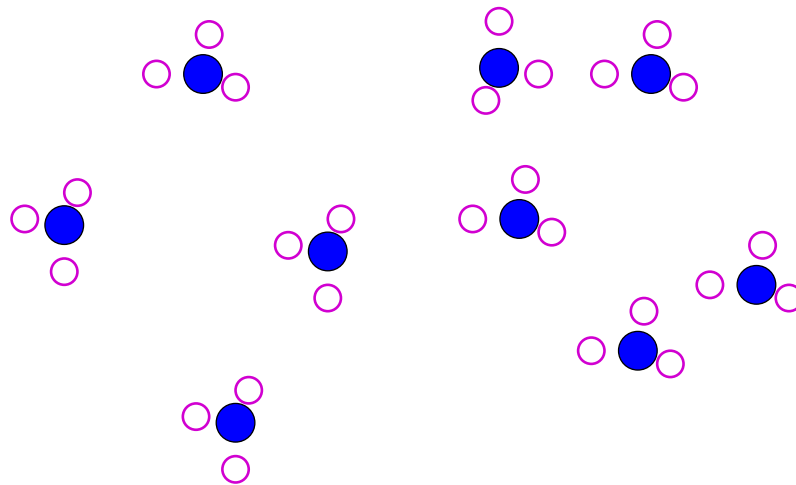
Understanding the FQHE requires a radically-new theory. You end up with **fractionally-charged** quasiparticles which have been **observed!**

For $\nu = 1/m$, m an integer, the ground state with n electrons is described by the Laughlin wave function:

$$\psi(z_1, z_2, \dots, z_n) \propto \prod_{i < j \leq n} (z_i - z_j)^m$$

where $z \equiv x + iy$.

For $m = 3$, pictorially



The filled circles are the electrons, the unfilled ones the zeros.

A **quasiparticle** at $z = z_0$ is described by the wave function

$$\psi_{excited}(z_0; z_1 \dots z_n) = \psi_{ground}(z_1, \dots z_n) \prod_i (z_i - z_0)$$

Physically, this corresponds to inserting a flux tube of strength $\Phi_0 = hc/e$.

In the presence of a flux tube, the wave function picks up a phase when moved from x_1 to x_2 :

$$\psi \rightarrow \psi e^{\frac{ie}{\hbar c} \int_{x_1}^{x_2} \vec{A} \cdot d\vec{l}}$$

Making a complete circuit,

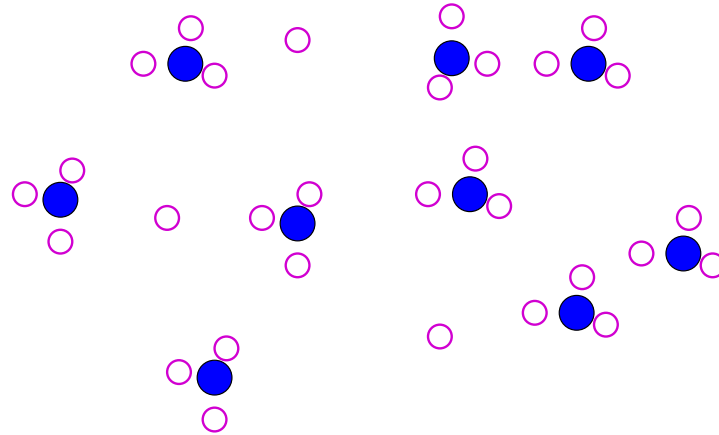
$$\begin{aligned} \oint \vec{A} \cdot d\vec{l} &= \int \int_{disc} \vec{B} \cdot \hat{n} dS \\ &= \Phi \end{aligned}$$

Thus under a complete circuit,

$$\psi \rightarrow \psi e^{\frac{ie}{\hbar c} \Phi}$$

even if the B -field is far away from the electron! **Aharonov-Bohm**

Thus quasiparticles result in extra zeroes in the wavefunction. For a three-particle state, we have



This state has charge $-e$, because we can add a single electron and get the ground state.

Quasiparticles have charge $-e/3$!!

The Laughlin wavefunction has been checked in many ways. Direct observations of the fractional charge rely on an important observation about these quasiparticles:

Bulk excitations are gapped

Edge excitations are gapless

This means that the effective theory of the quasiparticles is one-dimensional.

Halperin, 1982

This one-dimensional theory is understood. For the fractional QHE, these edge modes are described by a [chiral Luttinger model](#) (in particle physics language, the [massless Thirring model](#)). These theories are equivalent to a [massless free bosonic field](#) in 1+1 dimensions.

Wen, 1990

Probe this structure experimentally by building in a **point contact**:

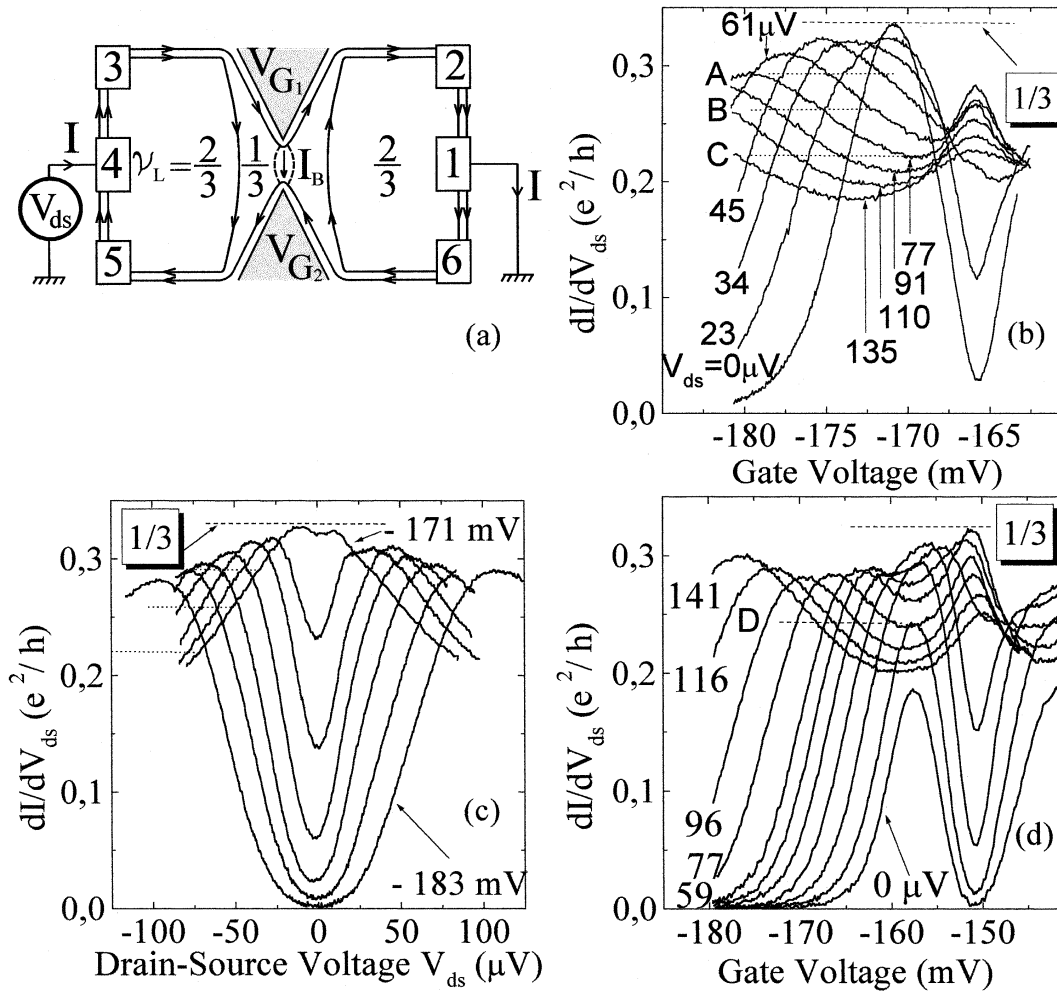


Figure 1

from Saminadayar et al, 1997

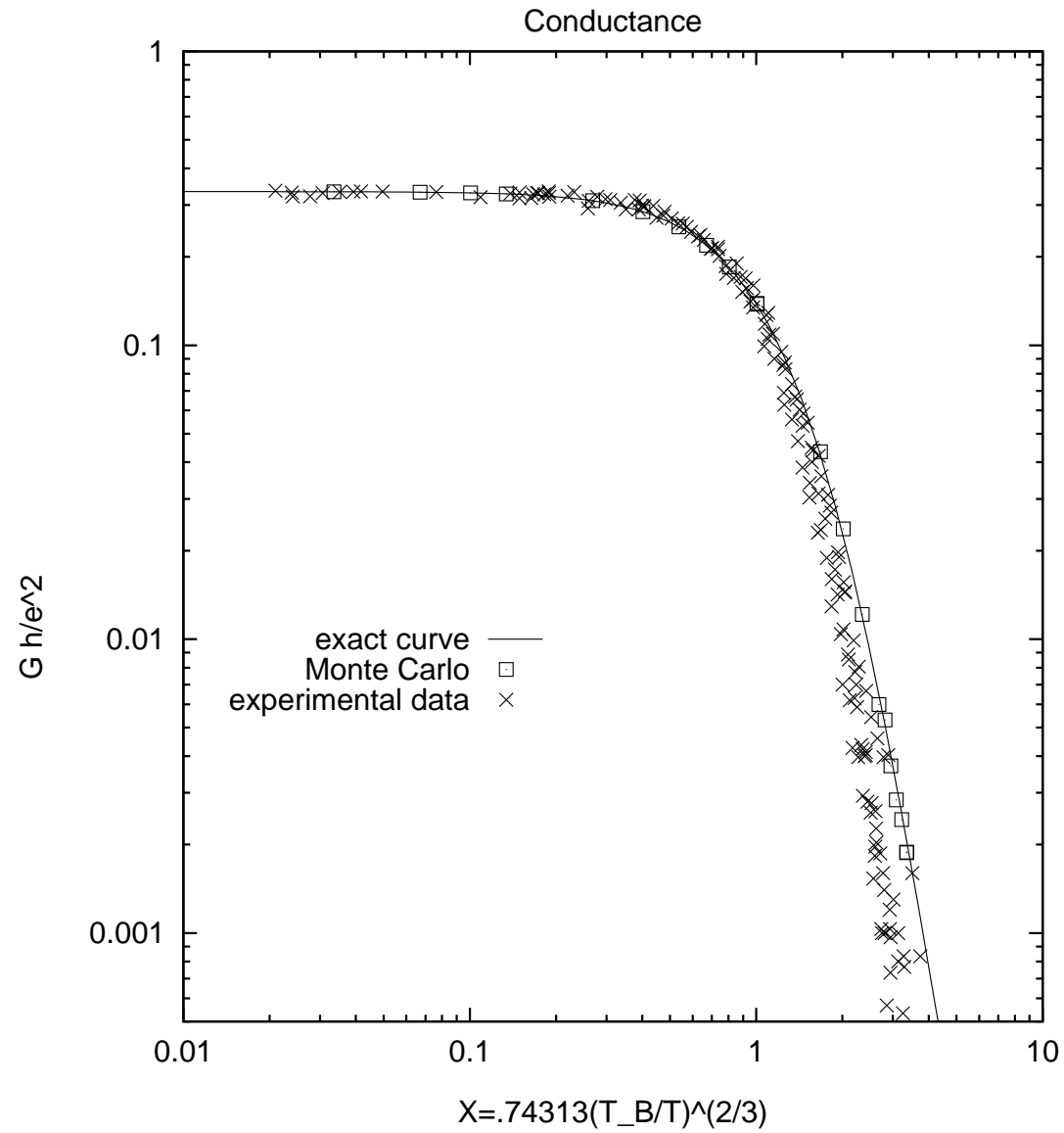
The experimentalist measure how the current across the contact (the backscattering) depends on the gate voltage (the size of the obstruction).

Theoretically, this is equivalent to a one dimensional field theory with an impurity ([boundary sine-Gordon model](#)).

Kane and Fisher, 1992

This theory can be solved!

Fendley, Ludwig and Saleur, 1994



data from [Milliken, Umbach and Webb, 1994](#)

Monte Carlo data from [Moon, Yi, Girvin, Kane and Fisher, 1994](#)

The fractional charge of the quasiparticle has been checked directly by several clever experiments.

Goldman and Su, 1995; Saminadayar et al, 1997; de Picciotto et al, 1997

A generic result for weak backscattering (small obstruction – independent events) is that the DC current noise

$$\langle (\Delta I)^2 \rangle = qI_B$$

where q is the charge of the object being backscattered, and I_B is the backscattered current.

Schottky, 1918

The full noise curve can be computed exactly as well.

Fendley and Saleur, 1996

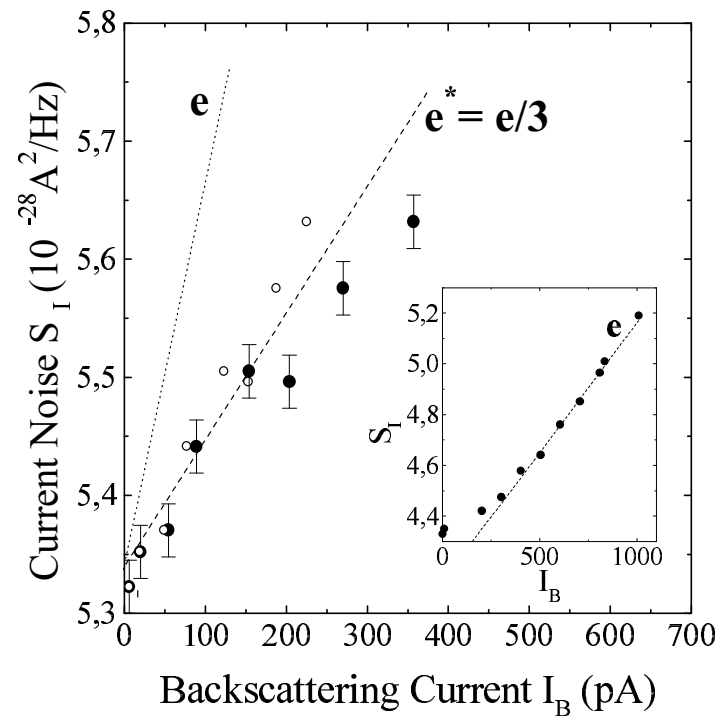
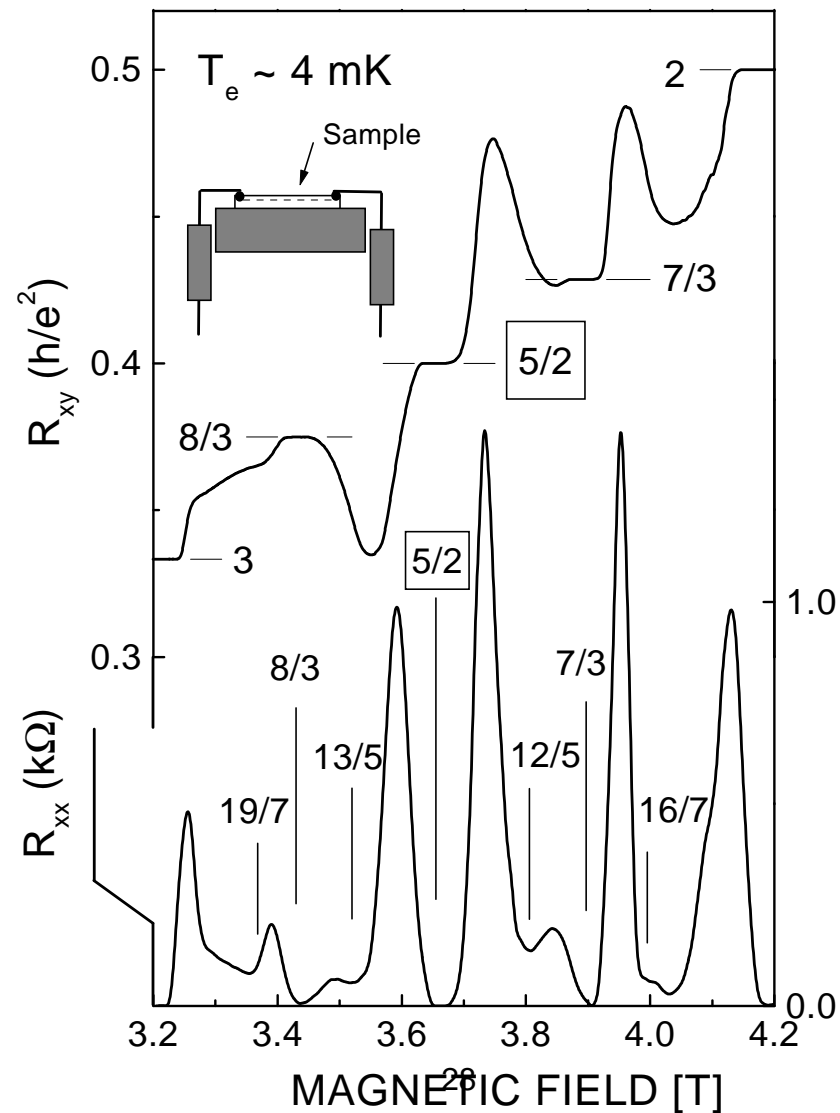


Figure 2

A charge of $1/5$ has been observed as well in the $\nu = 2/5$ FQHE.

A new frontier: there are now good observations of a plateau at $\nu = 5/2$ (even denominator).

Fig. 1, Pan et al



The only theory of the $5/2$ plateau consistent with the data (a spin-polarized state) predicts charge $1/4$ particles with non-Abelian statistics.

Moore and Read, 1991

In ordinary statistics in two dimensions, the wavefunction picks up a phase as two particles are exchanged. In three dimensions, this can only be ± 1 , but in two dimensions, any phase is possible. These are called anyons. The Laughlin quasiparticles are anyons.

An even stranger possibility is that if one has multiple particles, the phase picked up depends on the order in which the particles are exchanged. This is what happens to quasi-particles in the Moore-Read theory.

The other plateaus in the picture can be described by theories with non-Abelian statistics. The wavefunction is related to the parafermions of conformal field theory.

Read and Rezayi, 1997

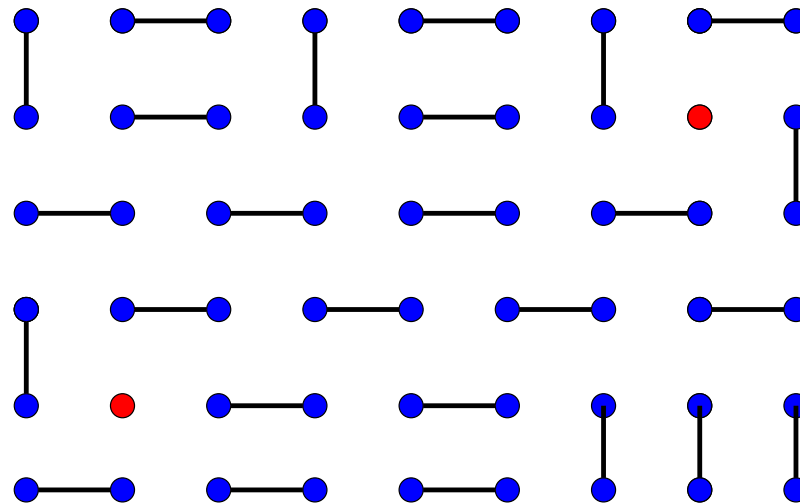
Where are we now?

In the last few years, the study of fractional charge has **intensified**.

Many theorists believe that the ideas of **spin-charge separation** and **fractional charge** are crucial to understanding high-temperature superconductivity.

The materials involved are anti-ferromagnetic, and many theorists have shown how fractional charge and spin-charge separation occur in various theories of antiferromagnets. In particular, Anderson proposed that high- T_c materials should possess an RVB phase, and that superconductivity occurs when the holons (instead of the electrons) condense into Cooper pairs.

To study this, look at the quantum dimer model:



Now, the degrees of freedom are the dimers, not electrons. A simple Hamiltonian to consider flips plaquettes with two dimers on them. One can show that this Hamiltonian as a large- N limit of certain antiferromagnetic models.

Rokhsar and Kivelson, 1988

Unfortunately, on the square lattice, the model orders (a spin-Peierls state) into columns of dimers, except at a single critical point, where the holons have algebraically decaying correlators.

Recent development:

On the [triangular lattice](#), there is an RVB phase!

[Moessner and Sondhi, 2001](#)

The correlators decay exponentially. In the ground state, the dimer-dimer correlator falls off as $e^{-2n/\xi}$ for n lattice spacings, where

$$\xi^{-1} = \frac{1}{4} \ln \left[\frac{\sqrt{2} + 3^{1/4}}{\sqrt{2} - 3^{1/4}} \right] = .83144 \dots$$

The holons are deconfined: the holon-holon correlator falls off exponentially to a value

$$0.14942924536134225401731517482693 \dots$$

only slightly below its nearest-neighbor value of $1/6$.

[Fendley, Moessner and Sondhi, 2002](#)

This computation requires using some very old Pfaffian techniques.

[Fisher and Stephenson, 1963](#)

We now have an explicit example of a model in an RVB phase. There are a number of proposals to observe this sort of phenomena in (frustrated) magnets as well as superconductors. There are even more speculative proposals that fractional charge will be useful for error correction in quantum computers.

In addition, the triangular-lattice quantum-dimer model gives an explicit realization of the \mathbb{Z}_2 lattice gauge theory approach to such physics.

Read and Sachdev, 1990

Senthil and Fisher, 2000-2003

many others...

Conclusions

1. Fractional charge is well understood theoretically.
2. Fractional charge exists in nature.
3. Fractional charge may end up explaining new physics in $2 + 1$ dimensions.

these transparencies at <http://rockpile.phys.virginia.edu>